

Neural Network-based Nonlinearity Estimation of Voltage Source Inverter for Synchronous Machine Drives

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Abstract—This paper investigates the concept of using a neural network (NN)-based approach for the nonlinearity estimation of voltage source inverter (VSI) in synchronous machine (SM) drives. The proposed scheme utilizes an NN with one hidden layer to model the VSI nonlinearity, accompanied by an adaptive law that ensures stability and bounded weights during the NN's update process. Assuming known stator flux linkages, the study primarily evaluates the feasibility of applying NN for this estimation. Simulation results from a 35 kW SM drive indicate that the proposed estimator successfully tracks the actual value of the VSI nonlinearity, demonstrating its efficacy.

Index Terms—Neural network (NN), online estimation, voltage source inverter (VSI), synchronous machine (SM), system identification

I. INTRODUCTION

Voltage source inverters (VSI) are critical in the operations of synchronous machine (SM) drives, enabling the conversion of DC to AC power with adjustable magnitude and frequency. This adaptability is fundamental for precise control across various applications, from industrial machinery to renewable energy systems. However, the inherent nonlinearity of VSI, arising from switching delays, dead time, and saturation effects, poses significant challenges, affecting control performance and system stability [1]. Accurate estimation and compensation of these nonlinearities are crucial for enhancing the performance and reliability of SM drives.

VSI nonlinearity is typically modeled in the abc frame, using sign functions of stator currents scaled by their amplitudes. Conventional methods for VSI nonlinearity estimation suggested identifying the amplitudes using recursive least square [2], heuristic optimization methods [3]–[5], current injection [6], [7], harmonic components [8]–[10], or adaptive method [11]. Nonetheless, empirical studies [12], [13] have demonstrated that VSI nonlinearities are more accurately represented by continuous functions with saturation. This insight

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indicates the necessity for a more generalized model that estimates the VSI nonlinearity directly.

This paper introduces a novel method that approaches VSI nonlinearity as a singular function estimated via a neural network (NN) with one hidden layer. This approach moves beyond traditional parameter-centric models to a more holistic representation that encompasses the continuity and saturation inherent in VSI nonlinearity. An adaptive law is derived for the NN's update process, ensuring stability and bounded weights. Assuming known stator flux linkages, the study primarily evaluates the feasibility of applying NN for this estimation. Simulation results from a 35 kW SM drive demonstrate the effectiveness of the proposed estimator in accurately tracking the actual VSI nonlinearity, showcasing its potential to significantly improve control accuracy and efficiency in SM drives.

II. PRELIMINARIES

A. SM Model

The SM can be expressed in the rotating dq frame as follows:

$$\begin{aligned}\dot{\lambda}_{dq}(t) &= \mathbf{v}_{dq}(t) - \omega_r \mathbf{J} \lambda_{dq}(t) - R_s \mathbf{i}_{dq}(t), \\ T_e(t) &= 1.5P \mathbf{i}_{dq}^T(t) \mathbf{J} \lambda_{dq}(t),\end{aligned}\quad (1)$$

with $\mathbf{z}_{dq} := [z_d \ z_q]^T$, $z = \lambda, v, i$, stator flux linkages λ_{dq} , stator voltages \mathbf{v}_{dq} , stator currents \mathbf{i}_{dq} , output torque T_e , number of pole pairs P , stator winding resistance R_s , rotation matrix $\mathbf{J} := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, and electrical rotor speed ω_r . In this paper, the following assumptions are made:

- The stator winding resistance R_s is accurately known.
- The iron loss in the electrical dynamics is neglected.
- The electrical rotor speed ω_r varies slowly compared to electrical quantities.

B. VSI Nonlinearity

The nonlinear effects of VSI, such as the dead time, turn-on delay, turn-off delay, and voltage drop of power devices, cause a mismatch Δ_z between the voltage references provided to the VSI \mathbf{v}_z^* and the actual output voltages produced by the VSI

v_z (i.e., $\Delta_z = v_z^* - v_z, z = dq$ or abc). Conventionally, such voltage error is modeled in the abc frame as follows [11]:

$$\Delta_{abc}(t) = \frac{V_{err}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} \text{sgn}(i_a(t)) \\ \text{sgn}(i_b(t)) \\ \text{sgn}(i_c(t)) \end{bmatrix}, \quad (2)$$

where Δ_{abc} is the voltage error in the abc frame; $i_a, i_b,$ and i_c are the a -, b -, and c -axis stator currents, respectively; and

$$V_{err} = \frac{T_{dead} + T_{on} - T_{off}}{T_s} V_{dc} + \frac{V_P + V_D}{2}, \quad (3)$$

where T_{dead} is the dead time, T_{on} is the turn-on delay, T_{off} is the turn-off delay, T_s is the sampling time, V_{dc} is the DC link voltage, V_P is the power device voltage drop, and V_D is the diode voltage drop, respectively. The voltage error is expressed in the rotating dq frame as follows:

$$\Delta_{dq}(t) = \frac{2}{3} \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \cos(\theta_r - \frac{2}{3}\pi) & -\sin(\theta_r - \frac{2}{3}\pi) \\ \cos(\theta_r - \frac{4}{3}\pi) & -\sin(\theta_r - \frac{4}{3}\pi) \end{bmatrix}^T \Delta_{abc}(t), \quad (4)$$

where θ_r is the electrical rotor position.

Conventional methods calculate the voltage error caused by the VSI nonlinearity using (2) or (4). However, this calculation demands accurate knowledge of VSI parameters to define V_{err} and cannot represent actual VSI nonlinearities with saturation effects; thus, online identification of VSI nonlinearity as a single function is necessitated.

III. NN-BASED VSI NONLINEARITY ESTIMATION

This section presents an NN-based VSI nonlinearity estimation scheme. It is assumed that information on the stator flux linkages is accurate to focus on the feasibility of using an NN for online estimation of VSI nonlinearity.

A. Estimator Design

The SM dynamics (1) can be expressed as the following state-space model:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(\omega_r)\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \Delta_{dq}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t), \end{aligned} \quad (5)$$

with state $\mathbf{x} := \lambda_{dq}$, input $\mathbf{u} := [(\mathbf{v}_{dq}^*)^T \quad \dot{\mathbf{i}}_{dq}^T]^T$, output $\mathbf{y} := \lambda_{dq}$, $\mathbf{A}(\omega_r) := -\omega_r \mathbf{J}$, $\mathbf{B} := [\mathbf{I}_2 \quad -R_s \mathbf{I}_2]$, and $\mathbf{C} := \mathbf{I}_2$. This state-space model is rewritten as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \bar{\mathbf{A}}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \epsilon_{dq}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t). \end{aligned} \quad (6)$$

with Hurwitz matrix $\bar{\mathbf{A}}$ such that the pair $(\mathbf{C}, \bar{\mathbf{A}})$ is observable and nonlinear term $\epsilon_{dq}(t) := \Delta_{dq}(t) + (\mathbf{A}(\omega_r) - \bar{\mathbf{A}})\mathbf{x}(t)$.

The VSI nonlinearity Δ_{dq} is modeled by continuous functions with saturation [12], [13]; thus, an NN with hyperbolic tangent function as activation functions can closely approximate the nonlinear term ϵ_{dq} including the VSI nonlinearity Δ_{dq} . The NN is defined as follows:

$$\epsilon_{dq}(t) = \mathbf{W}\sigma(\mathbf{V}\bar{\mathbf{x}}(t)), \quad (7)$$

with weight matrices $\mathbf{W} \in \mathbb{R}^{2 \times h}$ and $\mathbf{V} \in \mathbb{R}^{h \times 3}$, input vector $\bar{\mathbf{x}} = [\lambda_{dq}^T \quad \theta_r]^T$, number of nodes inside the hidden layer h , and hyperbolic tangent function σ . The weight matrices \mathbf{W} and \mathbf{V} are unknown matrices to be estimated online but are assumed to be bounded as

$$\|\mathbf{W}\|_F \leq W_M, \|\mathbf{V}\|_F \leq V_M. \quad (8)$$

Based on the rewritten model (6), the estimator is designed as follows:

$$\begin{aligned} \dot{\hat{\mathbf{x}}}(t) &= \bar{\mathbf{A}}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \hat{\epsilon}_{dq}(t), \\ \hat{\mathbf{y}}(t) &= \mathbf{C}\hat{\mathbf{x}}(t), \end{aligned} \quad (9)$$

with estimated state $\hat{\mathbf{x}}$, estimated output $\hat{\mathbf{y}}$, and estimated nonlinear term $\hat{\epsilon}_{dq} := \hat{\mathbf{W}}\sigma(\hat{\mathbf{V}}\hat{\mathbf{x}})$, where $\hat{\mathbf{W}}$ and $\hat{\mathbf{V}}$ are updated weight matrices by an adaptive law to approximate the ideal weight matrices \mathbf{W} and \mathbf{V} . The estimation error dynamics is obtained by subtracting (9) from (6) as follows:

$$\dot{\mathbf{e}}(t) = \bar{\mathbf{A}}\mathbf{e}(t) + \epsilon_{dq}(t) - \hat{\epsilon}_{dq}(t), \quad (10)$$

where $\mathbf{e} := \mathbf{x} - \hat{\mathbf{x}}$. The estimation error asymptotically converges to zero with a well-designed adaptive law that makes the term $\epsilon_{dq}(t) - \hat{\epsilon}_{dq}(t)$ negligibly small.

Such adaptive law is proposed based on the backpropagation as follows:

$$\begin{aligned} \dot{\hat{\mathbf{W}}} &= -\eta \frac{\partial G}{\partial \hat{\mathbf{W}}} - \rho \|e\| \hat{\mathbf{W}}, \\ \dot{\hat{\mathbf{V}}} &= -\eta \frac{\partial G}{\partial \hat{\mathbf{V}}} - \rho \|e\| \hat{\mathbf{V}}, \end{aligned} \quad (11)$$

with loss function $G := \frac{1}{2} e^T e$, learning rate η , and damping factor ρ . The first term in (11) is the backpropagation term and the second term is the exponential modification terms to guarantee the robustness of the adaptation [15]. This adaptive law is intended to reduce the estimation error \mathbf{e} by updating the weight matrices. Zero estimation error means the nonlinear term including the VSI nonlinearity is accurately modeled by the NN. The terms $\partial G / \partial \hat{\mathbf{W}}$ and $\partial G / \partial \hat{\mathbf{V}}$ are computed by employing the chain rule as follows:

$$\begin{aligned} \frac{\partial G}{\partial \hat{\mathbf{W}}} &= \frac{\partial G}{\partial e} \frac{\partial e}{\partial \hat{\mathbf{x}}} \frac{\partial \hat{\mathbf{x}}}{\partial \hat{\epsilon}_{dq}} \frac{\partial \hat{\epsilon}_{dq}}{\partial \hat{\mathbf{W}}}, \\ \frac{\partial G}{\partial \hat{\mathbf{V}}} &= \frac{\partial G}{\partial e} \frac{\partial e}{\partial \hat{\mathbf{x}}} \frac{\partial \hat{\mathbf{x}}}{\partial \hat{\epsilon}_{dq}} \frac{\partial \hat{\epsilon}_{dq}}{\partial \hat{\mathbf{V}}}. \end{aligned} \quad (12)$$

Because the term $\partial \hat{\mathbf{x}} / \partial \hat{\epsilon}_{dq}$ is difficult to compute, this is obtained using the static approximation (i.e. $\partial \hat{\mathbf{x}} / \partial \hat{\epsilon}_{dq} = 0$) as follows:

$$\frac{\partial \hat{\mathbf{x}}}{\partial \hat{\epsilon}_{dq}} = -\bar{\mathbf{A}}^{-1}. \quad (13)$$

Finally, the adaptive law is rewritten as follows:

$$\dot{\hat{\mathbf{W}}} = -\eta (e^T \bar{\mathbf{A}}^{-1})^T \hat{\sigma}^T - \rho \|e\| \hat{\mathbf{W}}, \quad (14)$$

$$\dot{\hat{\mathbf{V}}} = -\eta (e^T \bar{\mathbf{A}}^{-1} \hat{\mathbf{W}} (\mathbf{I}_h - \mathbf{\Pi}))^T \hat{\mathbf{x}}^T - \rho \|e\| \hat{\mathbf{V}}, \quad (15)$$

where $\hat{\sigma} := \sigma(\hat{\mathbf{V}}\hat{\mathbf{x}})$, \mathbf{I}_h is the identity matrix of size h , and $\mathbf{\Pi} := \text{diag}^2(\hat{\sigma})$.

The schematic diagram of the proposed NN-based VSI nonlinearity estimator is depicted in Fig. 1.

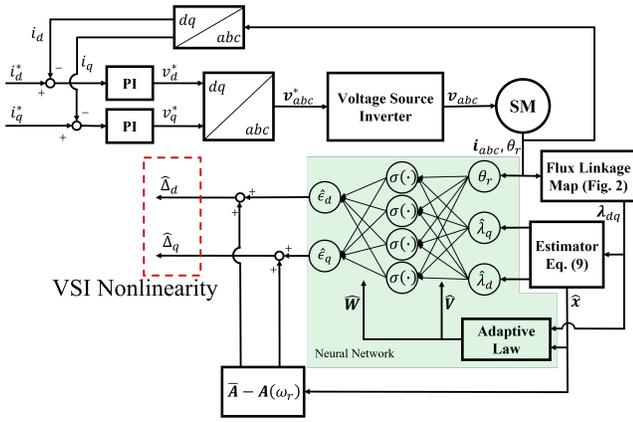


Fig. 1: Schematic diagram of the proposed NN-based VSI nonlinearity estimator

B. Stability Analysis

To verify the stability of the proposed estimator and the boundedness of the NN weights, the Lyapunov function is defined as follows:

$$L = \frac{1}{2} e^T P e + \frac{1}{2} \text{tr}(\tilde{W}^T \tilde{W}) + \frac{1}{2} \text{tr}(\tilde{V}^T \tilde{V}), \quad (16)$$

where $\tilde{W} := W - \hat{W}$, $\tilde{V} := V - \hat{V}$, and $P = P^T$ is a positive-definite matrix satisfying

$$\bar{A}^T P + P \bar{A} = -Q, \quad (17)$$

for some positive-definite matrix Q . The time-derivative of (16) is obtained as

$$\dot{L} = -\frac{1}{2} e^T Q e + e^T P \tilde{\epsilon} + \text{tr}(\tilde{W}^T \dot{\tilde{W}}) + \text{tr}(\tilde{V}^T \dot{\tilde{V}}), \quad (18)$$

where $\tilde{\epsilon} := \epsilon - \hat{\epsilon}$. In order to simplify the stability analysis, replace the update law of \hat{V} presented in (15) with

$$\dot{\hat{V}} = -\eta (e^T \bar{A}^{-1} \hat{W} (I_h - \Pi))^T \text{sgn}(\hat{x})^T - \rho \|e\| \hat{V}. \quad (19)$$

Substituting (14) and (19) into (18) yields the following inequality:

$$\begin{aligned} \dot{L} \leq & -\frac{1}{2} \lambda_{\min}(Q) \|e\|^2 + \|P\| (2W_M - \|\tilde{W}\|) \|e\| \\ & + \|\tilde{W}\| \|l\| \|e\| + \rho (W_M \|\tilde{W}\| - \|\tilde{W}\|^2) \|e\| \\ & + \|\tilde{V}\| (W_M + \|\tilde{W}\|) \|l\| \|e\| \\ & + \rho (V_M \|\tilde{V}\| - \|\tilde{V}\|^2) \|e\|, \end{aligned} \quad (20)$$

where $l := \eta \bar{A}^{-T}$ and $\lambda_{\min}(Q)$ is the minimum eigenvalue of matrix Q . The following condition on the estimation error e guarantees the negative semi-definiteness of \dot{L} :

$$\|e\| > R := \frac{2(2\|P\|W_M + (\rho - K_1^2)K_2^2 + (\rho - 1)K_3^2)}{\lambda_{\min}(Q)}, \quad (21)$$

where $K_1 := \|l\|/2$, $K_2 := (\|P\| + \|l\| + \rho W_M)/(2(\rho - K_1^2))$, and $K_3 := (\|l\|W_M + \rho V_M)/(2(\rho - 1))$. \dot{L} is negative semi-definite outside the ball with radius of R . Therefore, the proposed estimator is stable and has bounded weight matrices.

TABLE I
SPECIFICATIONS OF THE IPMSM DRIVE

Base speed	2000 RPM
Maximum torque	180 Nm
DC-link voltage (V_{dc})	325 V
Sampling time (T_s)	25 μ s
Number of pole pairs (P)	8
Stator resistance (R_s)	10.9 m Ω
Dead time (T_{dead})	1 μ s
Voltage drop of the diodes (V_D)	0.8 V
Voltage drop of the power devices (V_P)	0.5 V

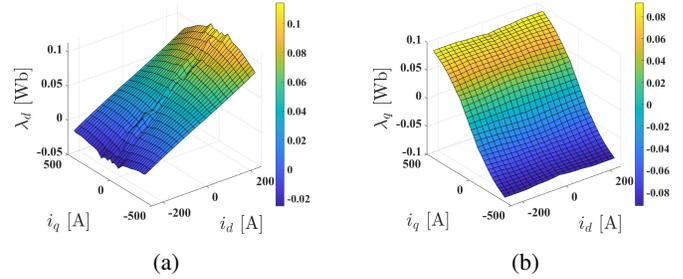


Fig. 2: Flux linkage maps of the IPMSM. (a) d -axis and (b) q -axis

IV. SIMULATION

The proposed NN-based VSI nonlinearity estimator was validated using MATLAB/SIMULINK simulation, built based on the ‘Three-phase PMSM Traction Drive’ example provided by MathWorks. A 35-kW interior permanent magnet SM (IPMSM) drive was used in the simulation, whose specifications and stator flux linkage maps are listed in Table I and shown in Figure 2. The turn-on delay T_{on} and turn-off delay T_{off} were neglected. The IPMSM drive was controlled by PI controllers to track the current references. A numerical reference generator presented in [14] was used to convert a torque command into the current references. The torque command linearly increased from 0 to 180 Nm during 0.02 s at a mechanical speed of 500 RPM. The estimator parameters were selected as follows: $h = 16$, $\eta = 12.5$, $\rho = 10^{-5}$, and $\bar{A} = \begin{bmatrix} -10^4 & -400 \\ 400 & -10^4 \end{bmatrix}$.

Figure 3 presents estimated states (i.e., stator flux linkages) of the proposed estimator, where the VSI nonlinearity is estimated by the NN and compensated, and estimated states calculated without the VSI nonlinearity compensation. Clearly, the estimated states with the compensation converged to the true states within 0.005 s. In contrast, the estimated states without the compensation did not converge to the true states. Figure 4 presents the norms of the NN’s weight matrices during the update process, which was verified to be bounded. The estimated VSI nonlinearity obtained by the proposed method is compared with the true value and the estimated value calculated by the conventional method [11], in Fig. 5. This comparison verified that the proposed method guaranteed to closely estimate the true value, while the conventional method

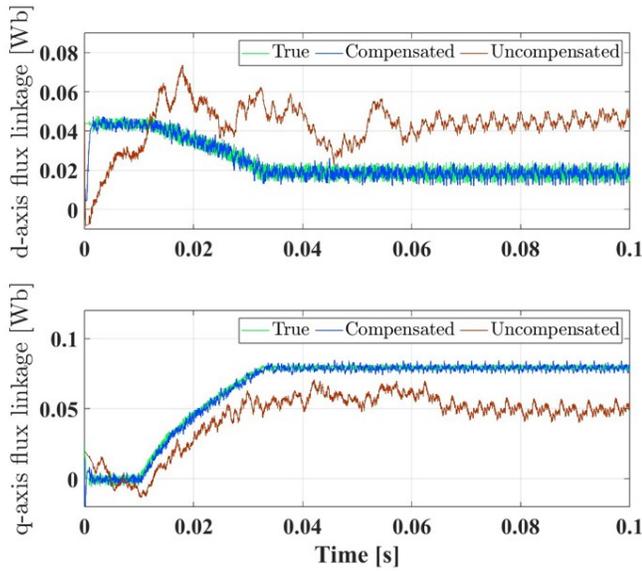


Fig. 3: True and estimated stator flux linkages.

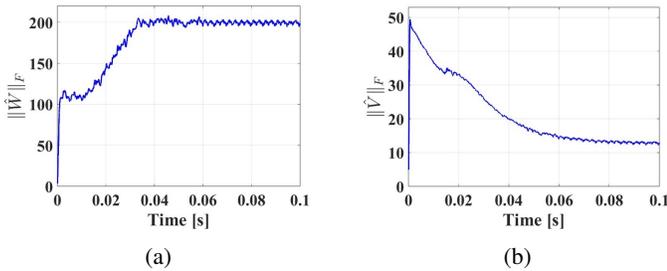


Fig. 4: Norms of weight matrices. (a) $\|W\|_F$ and (b) $\|V\|_F$

failed to model the true tendency due to using predefined sign functions. The computation time histogram of the proposed method is shown in Fig. 6. The computation time ranged from 5 to 6 μs , demonstrating the real-time capability of the proposed method.

V. CONCLUSION

This paper has presented an NN-based estimator for VSI nonlinearity. The primary contributions are twofold: (i) the development of an analytical expression for VSI nonlinearity using an NN, and (ii) the formulation of an adaptive law that ensures stability and maintains bounded weights within the NN. Simulation results involving a 35 kW IPMSM drive have confirmed that the proposed estimator can accurately track the true VSI nonlinearity while keeping the NN weights bounded. This study serves as a preliminary test to assess the feasibility of employing an NN to estimate VSI nonlinearity online, assuming accurate stator flux linkage information is available. Future research will aim to simultaneously estimate VSI nonlinearity and stator flux linkages, with experimental validation planned. One approach could involve integrating the proposed method with the extended state observer-based stator

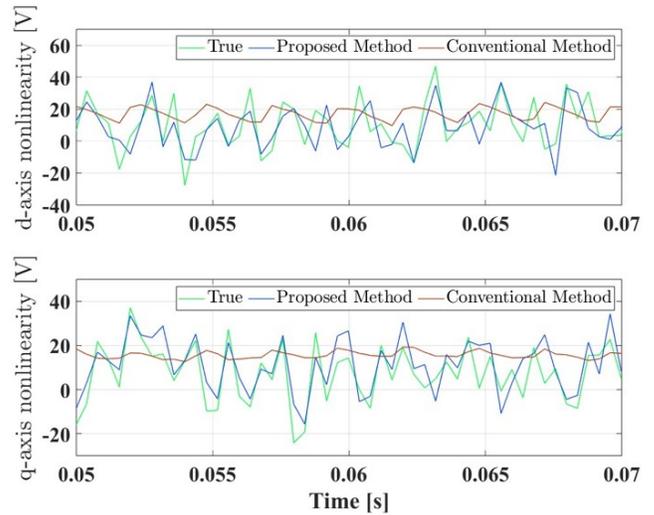


Fig. 5: Estimated VSI nonlinearity.

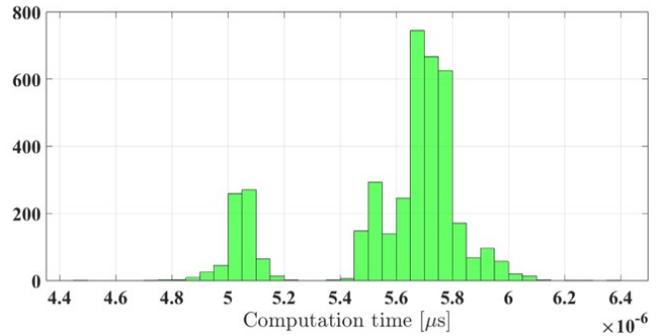


Fig. 6: Computation time of proposed adaptive law.

flux linkage estimator described in [16], rather than relying on actual stator flux linkage values.

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