LMI-based Neural Network Observer for State and Nonlinearity Estimation

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***Abstract*—This paper proposes a design method for linear matrix inequality (LMI)-based neural network (NN) observer gain in discrete-time domain. The proposed scheme employs an NN with a single hidden layer to approximate the lumped nonlinear term which includes uncertainties. A Lyapunov function is constructed to guarantee the stability of both the linear observer and the NN updates. The observer gain is determined by solving the LMI conditions, and the design is simplified by minimizing the number of tuning parameters, using a common gain structure for all vertices. Furthermore, designing an H_∞ observer can reduce the effect of NN approximation error and the measurement noise. The key advantages of the proposed method lie in its optimal LMI-based observer gain design, minimal tuning parameter requirement, and the capability to estimate both the system states and the lumped nonlinear term simultaneously. Simulation results indicate that the proposed method successfully tracks the actual states and the lumped nonlinear term and reduce the effects of NN approximation error and the measurement noise with comparison of the root mean square error (RMSE) values.**

***Index Terms*—neural networks (NN), linear matrix inequalities (LMI), H_∞ observers, discrete-time nonlinear systems**

I. Introduction

State estimation is important in the analysis and control of nonlinear systems. In particular, the problem of simultaneously estimating the system states and the nonlinearities in the discrete-time domain has received considerable attention over the past decades [1]–[3]. This task becomes significantly more challenging in the presence of system uncertainties, external disturbances, and measurement noise, all of which are common in practical engineering systems.

To address these challenges, various observer design techniques have been proposed, including Extended Kalman Filters [4]–[6], Sliding Mode Observers [7]–[9], and neural network-based observers [10]–[12]. In [10], the update law of the weight matrices using backpropagation, combined with an e-modification term to incorporate damping, was proposed. However, this method requires the selection of many tuning parameters and does not guarantee stability, as it only keeps the estimation error within a bounded region. Due to these factors, the system may exhibit a tendency to diverge over time in response to rapid changes in operating point. In [11], a gain design approach based on LMI was presented to estimate the system state and update the neural network weight. Although this method allows for the computation of optimal gains and demonstrates robustness against external factors such as measurement noise, it may exhibit divergence after discretization due to the formulation in the continuous-time domain. In addition, similar to previous approach, it involves a large number of tuning parameters, making it complex and highly sensitive to parameter selection.

This paper focuses on the design of a discrete-time observer using a neural network with a single hidden layer to estimate both the system states and the lumped nonlinear term, which includes uncertainties. A linear observer is augmented with an NN-based nonlinear estimator, and a Lyapunov-based approach is employed to ensure the stability of the overall estimation scheme.

The proposed scheme determines the observer gain by solving an LMI-based problem, which is the gain that satisfies the stability for all vertices of the system. The number of tuning parameters is minimized, which simplifies the implementation and reduces design complexity. In addition, a neural network-based observer design that guarantees H_∞ performance to reduce the impact of the approximation error or measurement noise in the proposed method. Simulation results validate the effectiveness of the proposed LMI-based NN observer to accurately track the states and the nonlinear term and reduce the effects of the NN approximation error and the measurement noise with comparing the root mean square error.

II. Problem Statement

The following equation represents a discrete-time system with unknown uncertainties and measurement noise:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Ff(x_k), \\ y_k &= Cx_k + n_k, \\ f(x_k) &= W_k\sigma(Nz_k) + \varepsilon_k, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the known input vector, $f(x_k) \in \mathbb{R}^r$ is the uncertainties of the system, $y \in \mathbb{R}^p$ is the output vector, and $n \in \mathbb{R}^p$ denotes the measurement noise vector, respectively. $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $F \in \mathbb{R}^{n\times r}$ are the known constant matrices, and the matrix A is a Hurwitz matrix. The uncertainties $f(x_k)$ is represented with a single hidden layer neural network form with the weight

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matrix $W \in \mathbb{R}^{r\times h}$, the diagonal normalization matrix $N \in \mathbb{R}^{h\times h}$, the input of neural network $z \in \mathbb{R}^{h}$, the neural network approximation error $\varepsilon \in \mathbb{R}^{r}$, and the activation function $\sigma(\cdot)$. The index k indicates the discrete-time step.

Assuming that the selected activation function $\sigma(\cdot)$ is bounded such that

$$-\infty < \sigma_{min} \leq \sigma(\cdot) \leq \sigma_{max} < \infty. \tag{2}$$

III. Neural Network-based Observer

This section presents an NN-based observer for estimating the state vector and the lumped nonlinear term.

A. Observer Design

Based on the model (1), the observer is designed as follow:

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + Bu_k + Ff(\hat{x}_k) + L(y_k - \hat{y}_k), \\ \hat{y}_k &= C\hat{x}_k, \\ f(\hat{x}_k) &= \hat{W}_k\sigma(N\hat{z}_k), \end{aligned} \tag{3}$$

with the estimated state vector $\hat{x}$, estimated output vector $\hat{y}$, the observer gain $L \in \mathbb{R}^{n\times p}$, and the nonlinear term $f(\hat{x}_k)$ where $\hat{W}$ is updated weight matrix by an adaptive law to approximate the ideal weight matrix W. The scheme of the estimated nonlinear term $f(\hat{x}_k)$ is depicted in Figure 1. The estimation error dynamics is obtained by subtracting (1) by (3) as follows:

$$\begin{aligned} \tilde{x}_{k+1} &= (A - LC)\tilde{x}_k + F\tilde{W}_k\sigma(\hat{z}_k) + \phi_k, \\ \tilde{y}_k &= C\tilde{x}_k + n_k, \end{aligned} \tag{4}$$

where the estimation error $\tilde{x}_k = x_k - \hat{x}_k$, and $\phi_k = F\hat{W}_k(\sigma(Nz_k) - \sigma(N\hat{z}_k)) + F\varepsilon_k - Ln_k$. To facilitate stability analysis, the error term $F\tilde{W}_k\sigma(N\hat{z}_k)$ in the error dynamics (4) is reformulated to $F\hat{\Sigma}_k\tilde{w}_k$ with

$$\begin{aligned} \hat{\Sigma}_k &= I_r \otimes \sigma(N\hat{z}_k)^T, \\ \tilde{w}_k &= \begin{bmatrix} \tilde{w}_1^T \\ \tilde{w}_2^T \\ \vdots \\ \tilde{w}_r^T \end{bmatrix}_k, \end{aligned} \tag{5}$$

where $\otimes$ denotes the Kronecker product, I_r is a $r \times r$ identity matrix, $\tilde{w}_i$ is the row vector for ith row in the weight matrix $\tilde{W}$.

The estimation error $\tilde{x}_k$ converges to zero with the adaptive law that makes the term $F\hat{\Sigma}_k\tilde{w}_k$ to zero. Such adaptive law is proposed based on the backpropagation as follows:

$$\hat{w}_{k+1} = \hat{w}_k - T_s\eta\frac{\partial J_k}{\partial \hat{w}_k}, \tag{6}$$

with loss function $J_k := \frac{1}{2}\tilde{y}_k^T\tilde{y}_k$, the sampling time T_s, and the learning rate η. Using the chain rule to solve (6), the adaptive law is represented as follows:

$$\hat{w}_{k+1} = \hat{w}_k + T_s\eta\hat{\Sigma}_k^T F^T(I_n - A)^{-T}C^T\tilde{y}_k. \tag{7}$$

To represent the error dynamics for weight vector w, the static approximation method(i.e. $w_{k+1} = w_k$) is used. Finally,

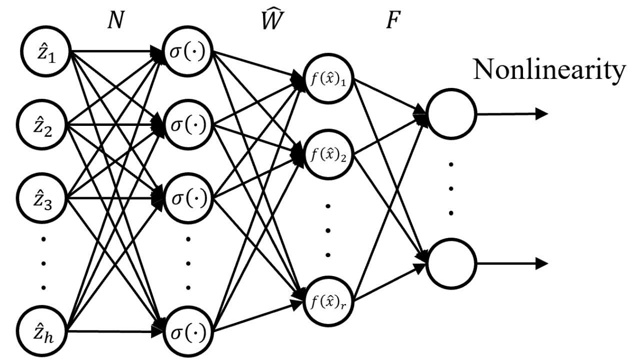

Fig. 1: Schematic diagram of the proposed NN estimator.

the error dynamics of the weight vector is represented as follows:

$$\tilde{w}_{k+1} = \tilde{w}_k - T_s\eta\hat{\Sigma}_k^T F^T(I_n - A)^{-T}C^T C\tilde{x}_k + \varphi_k, \tag{8}$$

with $\varphi_k = -T_s\eta\hat{\Sigma}_k^T F^T(I_n - A)^{-T}C^T n_k$.

B. Stability Analysis

The error dynamics (4) and (8) is reformulated the augmented state as follows:

$$\underbrace{\begin{bmatrix} \tilde{x}_{k+1} \\ \tilde{w}_{k+1} \end{bmatrix}}_{\xi_{k+1}} = \underbrace{\begin{bmatrix} A - LC & F\hat{\Sigma}_k \\ -T_s\eta\hat{\Sigma}_k^T F^T(I_n - A)^{-T}C^T C & I_h \end{bmatrix}}_{\mathcal{A}(\hat{\Sigma}_k)} \underbrace{\begin{bmatrix} \tilde{x}_k \\ \tilde{w}_k \end{bmatrix}}_{\xi_k} + \underbrace{\begin{bmatrix} \phi_k \\ \varphi_k \end{bmatrix}}_{\delta_k}, \tag{9}$$

with I_h is the $h \times h$ identity matrix.

The time-varying matrix $\mathcal{A}(\hat{\Sigma}_k)$ can be represented with affine parameter dependent model as follows:

$$\mathcal{A}(\hat{\Sigma}_k) = \mathcal{A}_0 + \mathcal{A}_1\Omega_k + \Omega_k^T\mathcal{A}_2, \tag{10}$$

where $\Omega_k = \begin{bmatrix} 0 & 0 \\ 0 & \hat{\Sigma}_k \end{bmatrix}$, $\mathcal{A}_0$ is the matrix with constant components of $\mathcal{A}(\hat{\Sigma}_k)$, $\mathcal{A}_1$ and $\mathcal{A}_2$ are the matrices where all elements are zero except for the matrix corresponding to $\hat{\Sigma}_k$. By the boundedness of $\sigma(\cdot)$ defined in (2), the matrix $\hat{\Sigma}_k$ is element-wise bounded as well. Since $\mathcal{A}(\cdot)$ is affine with respect to $\hat{\Sigma}_k$, stability of the system can be guaranteed over the entire bounded set if the stability condition is satisfied at all vertices of the uncertainty set. Using this approach, the observer gain L can be designed to satisfy 2^h LMI conditions corresponding to the vertices of the uncertainty set.

Theorem 1 presents a condition that guarantees the stability of the error dynamics (9).

Theorem 1: The estimation error dynamics (9) is stable if there exist matrices $P = P^T = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0$ and R of appropriate dimensions such that the following inequality is feasible:

$$\begin{bmatrix} \Gamma + (2\alpha - 1)P & \mathcal{A}(\hat{\Sigma}_k)^T P & \Pi^T \\ P\mathcal{A}(\hat{\Sigma}_k) & P & 0 \\ \Pi & 0 & -P_1 \end{bmatrix} \le 0,$$

$$\Gamma = \left[\begin{matrix} A^T P_1 A - A^T RC - C^T R^T A + \mathcal{A}_{21}^T P_2 \mathcal{A}_{21} \\ \mathcal{A}_{12}^T P_1 A - \mathcal{A}_{12}^T RC + P_2 \mathcal{A}_{21} \end{matrix} \right.$$
$$\left. \begin{matrix} A^T P_1 \mathcal{A}_{12} - C^T R^T \mathcal{A}_{12} + \mathcal{A}_{21}^T P_2 \\ \mathcal{A}_{12}^T P_1 \mathcal{A}_{12} + P_2 \end{matrix} \right],$$
$$\Pi = \begin{bmatrix} RC & 0 \end{bmatrix}$$
$$\mathcal{A}_{12} = F\hat{\Sigma}_k,$$
$$\mathcal{A}_{21} = -T_s \eta \hat{\Sigma}_k^T F^T (I_n - A)^{-T} C^T C. \tag{11}$$

When the inequality (11) is feasible, the linear observer gain L is given by $L = P_1^{-1} R$ and ensures that $\lim_{t\to\infty} \hat{x}_k = x_k$ exponentially.

Proof: To verify the stability of the proposed observer, the Lyapunov function is defined as follows:

$$V_k = \xi_k^T P \xi_k. \tag{12}$$

Now, it remains to prove that $\Delta V = V_{k+1} - V_k \le -2\alpha V_k$ for all $\xi_k \ne 0$:

$$\begin{aligned} \Delta V = \xi_k^T (\mathcal{A}^T P \mathcal{A} - P)\xi_k + \xi_k^T \mathcal{A}^T P \delta_k \\ + \delta_k^T P \mathcal{A} \xi_k + \delta_k^T P \delta_k. \end{aligned} \tag{13}$$

In order to satisfy the condition $\Delta V \le -2\alpha V_k$, the following inequality should be satisfied:

$$\begin{bmatrix} \xi_k \\ \delta_k \end{bmatrix}^T \mathcal{M} \begin{bmatrix} \xi_k \\ \delta_k \end{bmatrix} \le 0, \tag{14}$$

where $\mathcal{M} = \begin{bmatrix} \mathcal{A}^T P \mathcal{A} + (2\alpha - 1)P & \mathcal{A}^T P \\ P\mathcal{A} & P \end{bmatrix}$ and α is a fixed positive scalar that determines the convergence speed of the system (9).

Then, using the Schur complement, the notation $R = P_1 L$ and $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, the inequality (11) can be derived from the condition $\mathcal{M} \le 0$. Therefore, $\Delta V \le -2\alpha V_k$ for all $\xi_k \ne 0$. □

IV. H$_\infty$ Neural Network-based Observer

In this section, the robust H_∞ neural network observer is proposed to reduce the effects of δ_k such as the approximation error ε_k, and the measurement noise n_k. The observer gain with H_∞ gain γ is determined as follows:

$$\sup_{0<\|\delta_k\|_2<\infty} \frac{\|\xi_k\|_2}{\|\delta_k\|_2} \le \gamma, \tag{15}$$

where $\gamma > 0$ is an upper bound of H_∞ performance. This can be done by Theorem 2.

Theorem 2: For a positive scalar γ, the estimation error (9) converges if there exist matrix $P = P^T = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0$ and R of appropriate dimensions such that the following inequality is feasible:

$$\begin{bmatrix} \Gamma + (2\alpha - 1)P + I & \mathcal{A}(\hat{\Sigma}_k)^T P & \Pi^T \\ P\mathcal{A}(\hat{\Sigma}_k) & P - \gamma^2 I & 0 \\ \Pi & 0 & -P_1 \end{bmatrix} \le 0. \tag{16}$$

When the inequality (16) is feasible, the linear observer gain L is given by $L = P_1^{-1} R$ and ensures that $\lim_{t\to\infty} \hat{x}_k = x_k$ exponentially and minimizes the impact of NN approximation error or the measurement noise.

Proof: The H_∞ performance condition (15) is satisfied if the following inequality holds.

$$V_H = \Delta V + \|\xi_k\|^2 - \gamma^2 \|\delta_k\|^2 \le 0. \tag{17}$$

The inequality (17) becomes

$$\begin{aligned} \xi_k^T (\mathcal{A}^T P \mathcal{A} - P + I)\xi_k + \xi_k^T \mathcal{A}^T P \delta_k \\ + \delta_k^T P \mathcal{A} \xi_k + \delta_k^T (P - \gamma^2 I)\delta_k. \end{aligned} \tag{18}$$

The condition $V_H \le -2\alpha V_k$ can be represented in matrix form:

$$\begin{bmatrix} \xi_k \\ \delta_k \end{bmatrix}^T \bar{\mathcal{M}} \begin{bmatrix} \xi_k \\ \delta_k \end{bmatrix} \le 0, \tag{19}$$

where $\bar{\mathcal{M}} = \begin{bmatrix} \mathcal{A}^T P \mathcal{A} + (2\alpha - 1)P + I & \mathcal{A}^T P \\ P\mathcal{A} & P - \gamma^2 I \end{bmatrix}$.

Then, using the Schur complement, the notation $R = P_1 L$ and $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, the inequality (16) can be derived from the condition $\bar{\mathcal{M}} \le 0$. Therefore, $V_H \le -2\alpha V_k$ for all $\xi_k \ne 0$. □

Given the LMI condition in Theorem 2, the design of observer gain L can be achieved by convex optimization. In order to minimize the H_∞ norm (15), the following optimization problem using the LMIs,

$$\begin{aligned} &\min_{P,R} && \bar{\gamma} \\ &\text{subject to} && P > 0, \bar{\gamma} > 0, \text{ and (16)}, \end{aligned} \tag{20}$$

where $\bar{\gamma} = \gamma^2$.

V. Validation

The proposed LMI-based NN observer was validated in MATLAB/SIMULINK simulation. A permanent magnet SM (PMSM) compressor was used in simulation whose specifications are listed in Table I. The PMSM was controlled by PI controllers to track the current references. The PMSM is regulated to operate at a constant speed of 1800 RPM. The load torque is applied as a periodic input that varies proportionally with the rotational angle. The rotational speed dynamic system is as follows:

$$\dot{\omega}_m = -\frac{B_m}{J_m}\omega_m + \frac{1}{J_m}T_e - \frac{1}{J_m}T_L, \tag{21}$$

where ω_m is the rotational speed of the PMSM, T_e is the torque of the PMSM, and T_L is the load torque of the PMSM, respectively. However, uncertainty arises due to the

TABLE I
SPECIFICATIONS OF THE PMSM DRIVE

DC-link voltage (V_{dc})	150 V
Sampling time (T_s)	100 μs
Number of pole pairs (P)	4
Stator resistance (R_s)	1.1 Ω
d-axis Inductance (L_d)	8 mH
q-axis Inductance (L_q)	8 mH
Inertia (J_m)	4.44×10^{-4} kg·m^2
Friction (B_m)	0.005 Nm·s/rad
Known inertia ($\hat{J}_m$)	10^{-4} kg·m^2
Known friction ($\hat{B}_m$)	0.008 Nm·s/rad

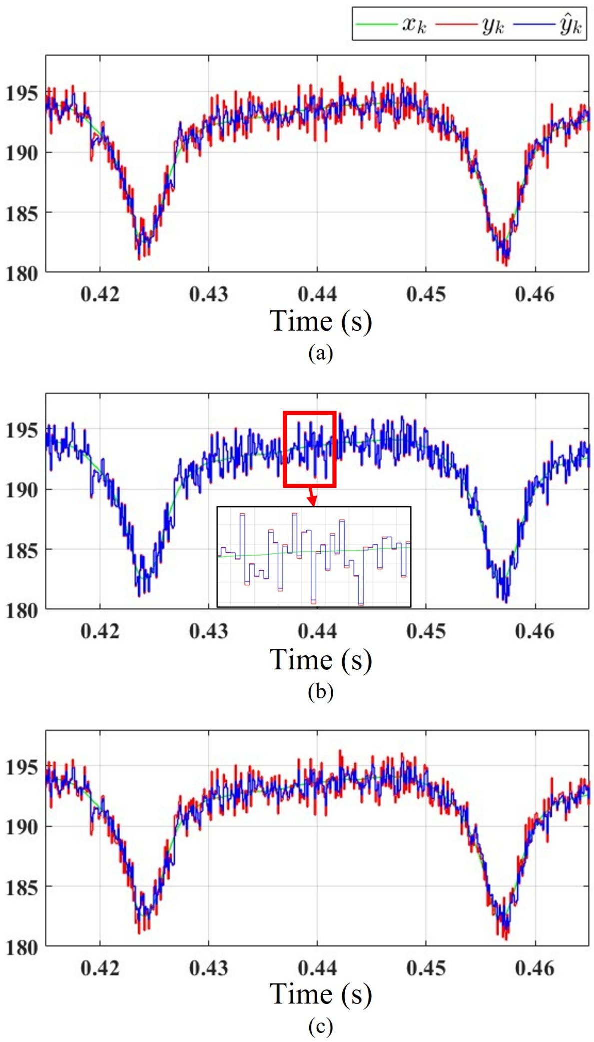

Fig. 2: Rotational speed of the PMSM compressor. (a) proposed method, (b) method in [10], and (c) method in [11].

lack of exact knowledge of the system parameters. The system parameters in (1) with Table I as follows:

$$x_k = \omega_m, \quad u_k = T_e^*, \\ A = 0.9920, \quad B = 1, \quad C = 1, \quad F = 1, \tag{22}$$

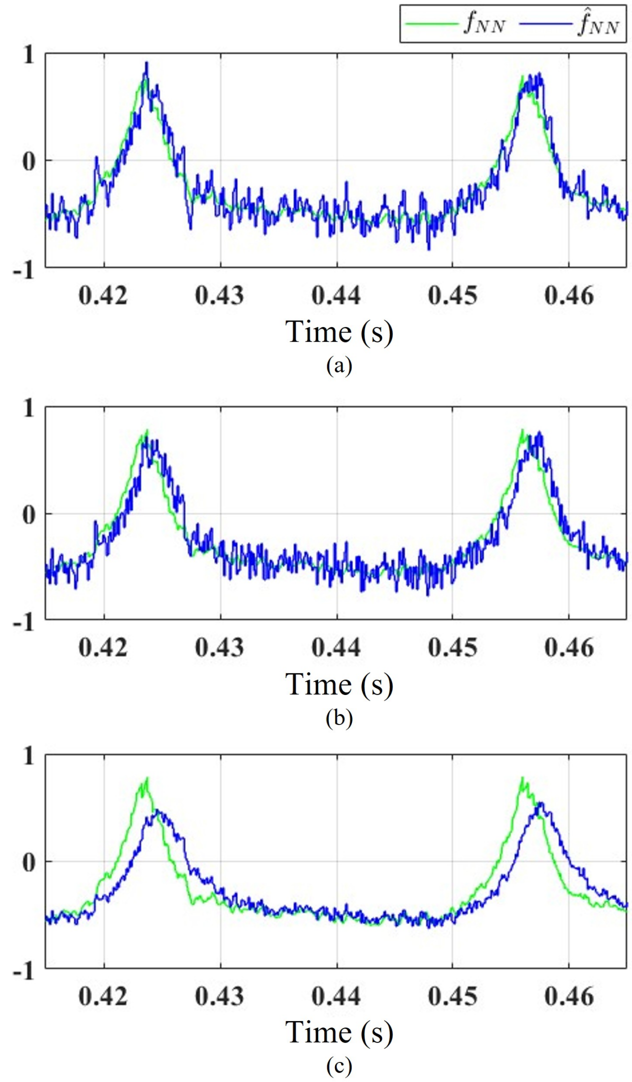

Fig. 3: Nonlinear uncertainty term. (a) proposed method, (b) method in [10], and (c) method in [11].

where T_e^* is the torque reference and $f(x_k)$ includes the load torque, difference between the torque reference and the torque of the PMSM, and parameter uncertainties such as inertia and friction. A numerical reference generator presented in [13] was used to convert a torque command into the current references. To evaluate the performance of the proposed method, it will be compared with the methods presented in [10] and [11]. The measurement noise is $n_k \sim \mathcal{N}(0, 0.01^2)$. The parameters for the validation of the proposed method were selected as follows: $\hat{z}_k = \begin{bmatrix}\hat{x}_k & y_k\end{bmatrix}^T$, $N = \begin{bmatrix}0.0053 & 0 \\ 0 & 0.0053\end{bmatrix}$, $\sigma(z_k) = \tanh(z_k) + 2$, $\alpha = 1.5$, $\eta = 0.7$, and the observer gain is obtained by solving (20) as $L = 0.4386$.

Figure 2 presents estimated the rotational speed of the PMSM compressor, where (a) is the proposed method in this

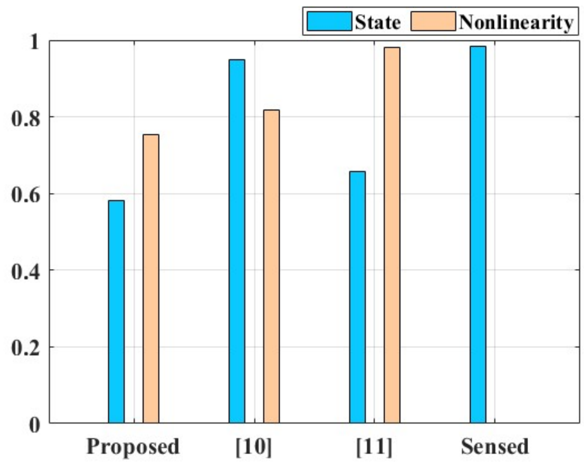

Fig. 4: Comparison of RMSE values for state and nonlinearity estimation results.

paper, (b) is the method in [10], and (c) is the method in [11]. The results in (a) and (c) indicate that the impact of the measurement noise is reduced, while the result in (b) shows similar estimation performance to the sensed rotational speed of the PMSM compressor due to the measurement noise.

Figure 3 presents the lumped nonlinear term $f(x_k)$ in (22) and the estimation results $f(\hat{x}_k)$. Among the results, (c) is the least affected by measurement noise; however, it exhibits the slowest convergence to the real nonlinearity. In contrast, the proposed method (a) demonstrates fast and accurate estimation performance, despite the presence of noise, which does not significantly affect the state estimation accuracy. In addition, case (b) shows divergence result as $t \to \infty$ due to the fast periodic application of the load torque and the estimation error remaining within a bounded region.

Figure 4 presents the comparison of root mean square error (RMSE) values for state and nonlinearity estimation during a 0.5 second simulation period. All values are normalized by the maximum value of each component. The proposed method demonstrates improved robustness to measurement noise, leading to more accurate state estimation and enhanced nonlinearity approximation. These results are attributed to the proposed method, which combines a backpropagation-based weight update to minimize the output error loss function to track the measurement values with an H_∞-based observer gain L designed to suppress the influence of the measurement noise and to be satisfied for all vertices. The results of all methods yield lower RMSE values compared to the directly sensed speed y_k, demonstrating robustness against measurement noise.

VI. Conclusion

This paper has presented an LMI-based neural network observer for the estimation of both the system states and nonlinear terms, with reduced sensitivity to measurement noise and other uncertainties in the discrete-time domain. A design method for the weight update rule based on the backpropagation, along with the observer gain derived by solving the LMI problem for all vertices is proposed. Furthermore, H_∞ observer is proposed to account for NN approximation error and measurement noise, aiming to minimize their effects. The effectiveness of the proposed method is validated through PMSM compressor simulation results. Future work will focus on experimental validation, potential extension to MIMO nonlinear systems, and the development of practical guidelines for tuning observer parameters such as learning rate.

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