All-Wheel Steering Vehicle Control Based on Contraction Theory with Neural Network

IEEE International Conference on Advanced Motion Control (AMC) 2026

Myeongseok Ryu[1] and Kyunghwan Choi[1]

[1]Mobility Intelligence and Control Laboratory (MIC Lab)
CCS Graduate School of Mobility
Korea Advanced Institute of Science and Technology (KAIST)

2026-03-09

All-Wheel Steering (AWS) provides enhanced maneuverability and stability by allowing independent control of the front and rear wheels, rather than front-wheel steering (FWS) alone.

- **Reverse-phase (at low speed):** Steering rear wheels in the opposite direction
  $\rightarrow$ enhanced maneuverability by reducing turning radius.
- **In-phase (at high speed):** Steering rear wheels in the same direction
  $\rightarrow$ enhanced stability by reducing slip angle.
- Many control approaches have been applied, e.g., MPC [1], SMC [2], and robust control [3].

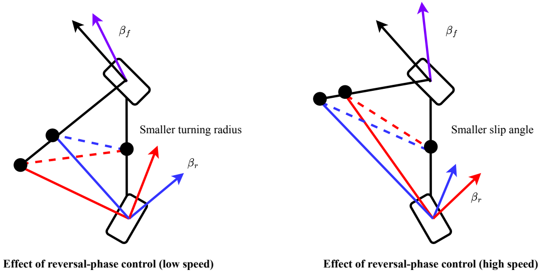


Figure: Effectiveness of All-Wheel Steering (AWS [blue], FWS [red]).

Contraction Theory is a powerful tool for analyzing the stability of nonlinear systems by examining differential dynamics, providing insights into the behavior of trajectories (not points) and their convergence properties [4].

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, t) \quad \rightarrow \quad \text{diff. dyn. } \frac{\mathrm{d}}{\mathrm{d}t}\delta\boldsymbol{x} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\delta\boldsymbol{x}. \tag{1}$$

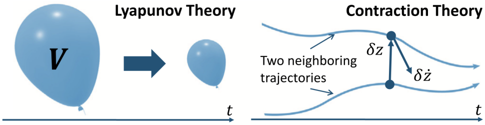


Figure: Lyapunov theory versus contraction theory [5].

Contraction Theory is a powerful tool for analyzing the stability of nonlinear systems by examining differential dynamics, providing insights into the behavior of trajectories (not points) and their convergence properties [4].

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, t) \quad \rightarrow \quad \text{diff. dyn. } \frac{\mathrm{d}}{\mathrm{d}t}\delta\boldsymbol{x} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\delta\boldsymbol{x}. \tag{1}$$

- If there exists positive definite Contraction Metric $\boldsymbol{M}$ such that $\frac{\mathrm{d}}{\mathrm{d}t}V \leq -\alpha V$, the system is contracting.
- $\delta\boldsymbol{x}$ shrinks exponentially in the Riemannian space defined by $\boldsymbol{M}$ (exponential convergence of trajectories).

$$\text{Lya. func. } V = \delta\boldsymbol{x}^\top \boldsymbol{M}(\boldsymbol{x}, t)\delta\boldsymbol{x}, \quad \text{cond. for contraction } \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M} + 2\,\mathrm{sym}\left(\boldsymbol{M}\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right) \leq -2\alpha\boldsymbol{M} \tag{2}$$

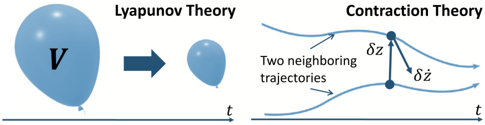


Figure: Lyapunov theory versus contraction theory [5].

Contraction Theory is a powerful tool for analyzing the stability of nonlinear systems by examining differential dynamics, providing insights into the behavior of trajectories (not points) and their convergence properties [4].

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, t) \quad \rightarrow \quad \text{diff. dyn. } \frac{\mathrm{d}}{\mathrm{d}t}\delta\boldsymbol{x} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\delta\boldsymbol{x}. \tag{1}$$

- If there exists positive definite Contraction Metric $\boldsymbol{M}$ such that $\frac{\mathrm{d}}{\mathrm{d}t}V \leq -\alpha V$, the system is contracting.
- $\delta\boldsymbol{x}$ shrinks exponentially in the Riemannian space defined by $\boldsymbol{M}$ (exponential convergence of trajectories).

$$\text{Lya. func. } V = \delta\boldsymbol{x}^\top \boldsymbol{M}(\boldsymbol{x}, t)\delta\boldsymbol{x}, \quad \text{cond. for contraction } \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M} + 2\,\mathrm{sym}\left(\boldsymbol{M}\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right) \leq -2\alpha\boldsymbol{M} \tag{2}$$

- **Exponential Convergence**: Trajectories globally exponentially converge to each other regardless of initial conditions.
- **Control Design**: Owing to nature of differential dynamics, LPV control design methods can be applied.

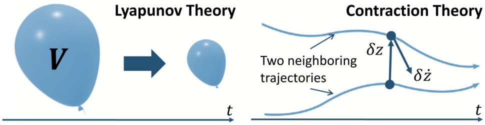


Figure: Lyapunov theory versus contraction theory [5].

Challenges in Contraction Theory Based Control Design

Uncertain parameters degrades the performance, as they rely on accurate system models to ensure contraction.

Challenges in Contraction Theory Based Control Design

Uncertain parameters degrades the performance, as they rely on accurate system models to ensure contraction.

Neural networks (NNs) can approximate any smooth function [6].

- NNs can be employed to compensate for model uncertainties.
- Online learning capabilities of NNs and closed-loop stability are ensured using Lyapunov stability analysis [7].

Challenges in Contraction Theory Based Control Design

Uncertain parameters degrades the performance, as they rely on accurate system models to ensure contraction.

Neural networks (NNs) can approximate any smooth function [6].

- NNs can be employed to compensate for model uncertainties.
- Online learning capabilities of NNs and closed-loop stability are ensured using Lyapunov stability analysis [7].

Neural networks can be integrated with contraction theory based control to compensate for model uncertainties by learning online.

MIC Lab
Mobility Intelligence and Control Laboratory

Contributions of this work:

Contributions of this work:

1. We propose AWS based on contraction theory integrated with neural networks.

Contributions of this work:

1. We propose AWS based on contraction theory integrated with neural networks.
2. Contraction metrics are obtained by solving LMI problems.

Contributions of this work:

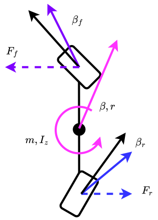
1. We propose AWS based on contraction theory integrated with neural networks.
2. Contraction metrics are obtained by solving LMI problems.
3. Parameter uncertainties are compensated by NNs.

Lateral Dynamics: [8]

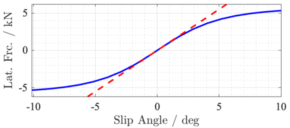
- 2-DOF bicycle model (3).
- β: sideslip angle, r: yaw rate.
- δ_f: front steering angle, δ_r: rear steering angle.
- Tire forces contains uncertain parameters (cornering stiffness C_f, C_r).

$$\begin{cases} mv_x(\frac{\mathrm{d}}{\mathrm{d}t}\beta + r) = F_f + F_r, \\ I_z\frac{\mathrm{d}}{\mathrm{d}t}r = F_f l_f - F_r l_r, \end{cases} \tag{3}$$

$$\begin{cases} F_f = C_f\left(\beta + \frac{l_f r}{v_x} - \delta_f\right), \\ F_r = C_r\left(\beta - \frac{l_r r}{v_x}\right). \end{cases} \tag{4}$$



Bicycle Vehicle Model



Tire Characteristic

Figure: Vehicle model and tire forces.

System Modeling:

- State vector $\boldsymbol{x} = (\beta, r)^\top$, control input $\boldsymbol{u} = (\delta_f, \delta_r)^\top$.
- Uncertain parameters $\boldsymbol{\pi} = (C_f, C_r)^\top$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\pi}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})\boldsymbol{u} + \boldsymbol{d}, \qquad (5)$$

MIC Lab
Mobility Intelligence and Control Laboratory

System Modeling:

- State vector $\boldsymbol{x} = (\beta, r)^\top$, control input $\boldsymbol{u} = (\delta_f, \delta_r)^\top$.
- Uncertain parameters $\boldsymbol{\pi} = (C_f, C_r)^\top$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\pi}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})\boldsymbol{u} + \boldsymbol{d}, \tag{5}$$

Desired Trajectory:

- Simulate linear 2-DOF reference model with nominal parameters $\boldsymbol{\pi}_n$.
- Assume ideal control input $\boldsymbol{u}_d$ is given in advance.
- Entries of worst-case input matrix $\underline{\boldsymbol{g}}$ are lower bounds of $\boldsymbol{g}$'s entries.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_d = \boldsymbol{f}(\boldsymbol{x}_d, \boldsymbol{\pi}_n) + \underline{\boldsymbol{g}}\boldsymbol{u}_d, \tag{6}$$

System Modeling:

- State vector $\boldsymbol{x} = (\beta, r)^\top$, control input $\boldsymbol{u} = (\delta_f, \delta_r)^\top$.
- Uncertain parameters $\boldsymbol{\pi} = (C_f, C_r)^\top$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\pi}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})\boldsymbol{u} + \boldsymbol{d}, \tag{5}$$

Desired Trajectory:

- Simulate linear 2-DOF reference model with nominal parameters $\boldsymbol{\pi}_n$.
- Assume ideal control input $\boldsymbol{u}_d$ is given in advance.
- Entries of worst-case input matrix $\underline{\boldsymbol{g}}$ are lower bounds of $\boldsymbol{g}$'s entries.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_d = \boldsymbol{f}(\boldsymbol{x}_d, \boldsymbol{\pi}_n) + \underline{\boldsymbol{g}}\boldsymbol{u}_d, \tag{6}$$

LPV Formulation using State Dependent Coefficients (SDC):

- $\mathbb{A} := \mathbb{A}(\boldsymbol{x}, \boldsymbol{x}_d) = \int_0^1 \left[\frac{\partial \overline{\boldsymbol{f}}}{\partial \boldsymbol{x}}\right] (c\boldsymbol{x} + (1-c)\boldsymbol{x}_d)\,\mathrm{d}c$,
  where $\overline{\boldsymbol{f}} := \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{\pi}_n) - \boldsymbol{f}(\boldsymbol{x}_d, \boldsymbol{\pi}_n) + \left(\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi}_n) - \underline{\boldsymbol{g}}\right)\boldsymbol{u}_d$,
  see [9, Lemma 1].
- $\boldsymbol{\delta}(\boldsymbol{x}, \boldsymbol{\pi}, \boldsymbol{\pi}_n, \boldsymbol{u}_d)$ is lumped uncertainty.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \mathbb{A}(\boldsymbol{x} - \boldsymbol{x}_d) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})(\boldsymbol{u} - \boldsymbol{u}_d) + \boldsymbol{\delta}, \tag{7}$$

Control Objectives:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \mathbb{A}(\boldsymbol{x}-\boldsymbol{x}_d) + \boldsymbol{g}(\boldsymbol{x},\boldsymbol{\pi})(\boldsymbol{u}-\boldsymbol{u}_d) + \boldsymbol{\delta}, \tag{8}$$

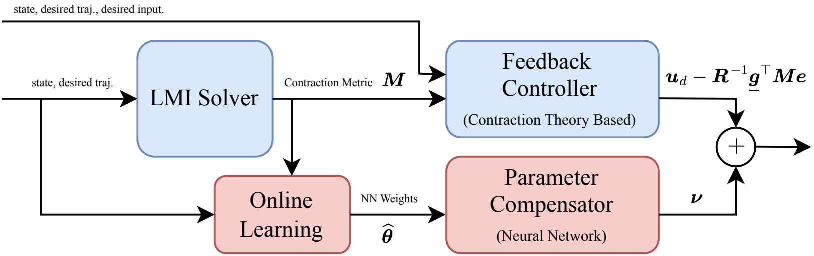


Figure: Diagram of Proposed Controller.

Control Objectives:

- Design a control law $\boldsymbol{u}$ to track the desired trajectory $\boldsymbol{x}_d$ by making error dynamics (8) contracting.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \mathbb{A}(\boldsymbol{x}-\boldsymbol{x}_d) + \boldsymbol{g}(\boldsymbol{x},\boldsymbol{\pi})(\boldsymbol{u}-\boldsymbol{u}_d) + \boldsymbol{\delta}, \tag{8}$$

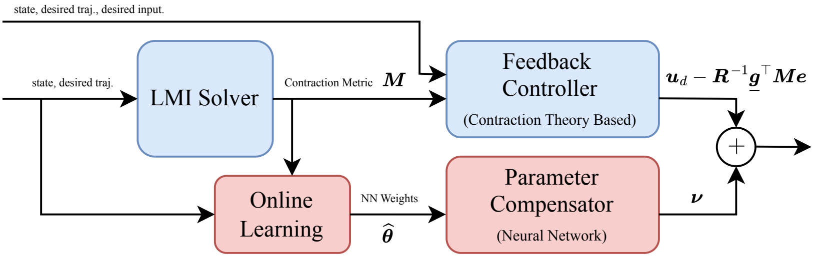


Figure: Diagram of Proposed Controller.

Control Objectives:

- Design a control law $\boldsymbol{u}$ to track the desired trajectory $\boldsymbol{x}_d$ by making error dynamics (8) contracting.
- Ensure the control law is robust against parameter uncertainty considering the worst-case input matrix $\underline{\boldsymbol{g}}$.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \mathbb{A}(\boldsymbol{x} - \boldsymbol{x}_d) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})(\boldsymbol{u} - \boldsymbol{u}_d) + \boldsymbol{\delta}, \tag{8}$$

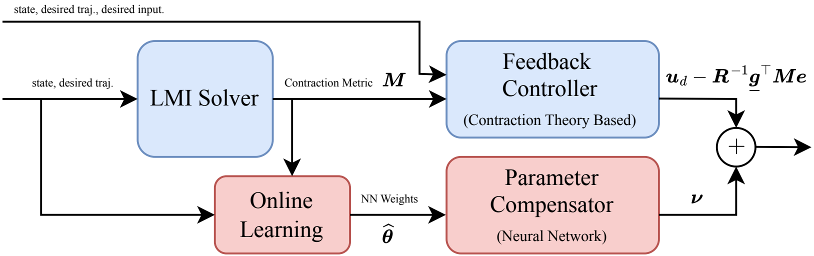


Figure: Diagram of Proposed Controller.

Control Objectives:

- Design a control law $\boldsymbol{u}$ to track the desired trajectory $\boldsymbol{x}_d$ by making error dynamics (8) contracting.
- Ensure the control law is robust against parameter uncertainty considering the worst-case input matrix $\underline{\boldsymbol{g}}$.
- Compensate for the lumped uncertainty $\boldsymbol{\delta}$ using a neural network with online learning capability.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \mathbb{A}(\boldsymbol{x} - \boldsymbol{x}_d) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{\pi})(\boldsymbol{u} - \boldsymbol{u}_d) + \boldsymbol{\delta}, \tag{8}$$

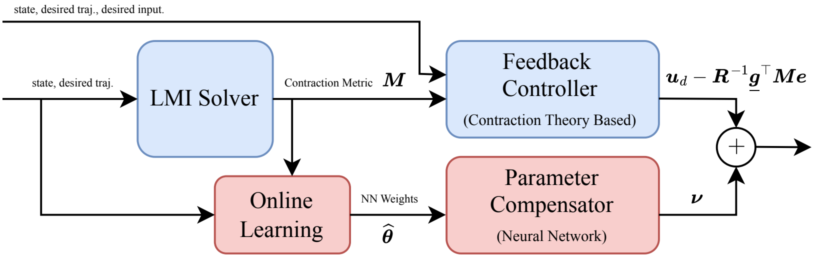


Figure: Diagram of Proposed Controller.

Contraction Theory Based Control Design:

- Feedback control law using contraction metric $\boldsymbol{M}$.
- Contraction metric $\boldsymbol{M}$ should satisfy (11) to ensure the error dynamics (8) is contracting.
- $\boldsymbol{R}$ is weighting matrix for control input and $\boldsymbol{\nu}$ is auxiliary control input (NN) for the lumped uncertainty $\boldsymbol{\delta}$.

$$\boldsymbol{u} := \boldsymbol{u}_d - \boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\boldsymbol{e} + \boldsymbol{\nu}, \tag{9}$$

which lead to the following error dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \left(\mathbb{A} - \boldsymbol{g}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\right)\boldsymbol{e} + \boldsymbol{g}\boldsymbol{\nu} + \boldsymbol{\delta}, \tag{10}$$

MIC Lab Mobility Intelligence and Control Laboratory

Contraction Theory Based Control Design:

- Feedback control law using contraction metric $\boldsymbol{M}$.
- Contraction metric $\boldsymbol{M}$ should satisfy (11) to ensure the error dynamics (8) is contracting.
- $\boldsymbol{R}$ is weighting matrix for control input and $\boldsymbol{\nu}$ is auxiliary control input (NN) for the lumped uncertainty $\boldsymbol{\delta}$.

$$\boldsymbol{u} := \boldsymbol{u}_d - \boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\boldsymbol{e} + \boldsymbol{\nu}, \tag{9}$$

which lead to the following error dynamics:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{e} = \left(\mathbb{A} - \boldsymbol{g}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\right)\boldsymbol{e} + \boldsymbol{g}\boldsymbol{\nu} + \boldsymbol{\delta}, \tag{10}$$

Condition for Contraction and Steady State Error:

- related theorem is omitted for brevity, see [4].
- Minimizing condition number $\chi := \mathrm{cond}(\boldsymbol{M})$ can reduce the steady state error bound.

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{M} + 2\,\mathsf{sym}(\boldsymbol{M}\mathbb{A}) - 2\,\mathsf{sym}(\boldsymbol{M}\boldsymbol{g}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}) \preceq -2\alpha\boldsymbol{M}. \tag{11}$$

$$\lim_{t\to\infty}\|\boldsymbol{e}(t)\| = \sup_{\boldsymbol{x}\in\Omega_x,\boldsymbol{\pi},\boldsymbol{\pi}_n\in\Omega_\pi}\|\boldsymbol{g}\boldsymbol{\nu} + \boldsymbol{\delta}\|\frac{\sqrt{\chi}}{\alpha}, \tag{12}$$

LMI formulation and Consideration of Worst-Case Input Matrix:

- By multiplying $\boldsymbol{M}^{-1} = \boldsymbol{W}$, the contraction condition (11) can be reformulated as the following LMI.
- Considering the worst-case input matrix $\underline{\boldsymbol{g}}$, the LMI condition can be relaxed to (13).

$$-\frac{\boldsymbol{W}-\boldsymbol{W}_{\mathrm{pre}}}{T_s} + 2\,\mathrm{sym}(\mathbb{A}\boldsymbol{W}) + 2\alpha\boldsymbol{W} \preceq 2\,\mathrm{sym}\left(\underline{\boldsymbol{g}}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^{\top}\right) \preceq 2\,\mathrm{sym}\left(\boldsymbol{g}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^{\top}\right). \tag{13}$$

LMI formulation and Consideration of Worst-Case Input Matrix:

- By multiplying $\boldsymbol{M}^{-1} = \boldsymbol{W}$, the contraction condition (11) can be reformulated as the following LMI.
- Considering the worst-case input matrix $\underline{\boldsymbol{g}}$, the LMI condition can be relaxed to (13).

$$-\frac{\boldsymbol{W}-\boldsymbol{W}_{\text{pre}}}{T_s} + 2\,\text{sym}(\mathbb{A}\boldsymbol{W}) + 2\alpha\boldsymbol{W} \preceq 2\,\text{sym}\left(\underline{\boldsymbol{g}}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top\right) \preceq 2\,\text{sym}\left(\boldsymbol{g}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top\right). \tag{13}$$

Final Optimization Problem to obtain M:

- By solving (14), $\boldsymbol{M} = \boldsymbol{W}^{-1}$ can be obtained for the feedback control law.
- λ can be used to balance condition number (steady state error) and magnitude of $\boldsymbol{M}$ (control effort).

$$\min_{\overline{\boldsymbol{W}},\chi,\mu} \chi + \lambda\mu$$

$$\text{s.t.} \begin{cases} \boldsymbol{I}_n \preceq \overline{\boldsymbol{W}} \preceq \chi\boldsymbol{I}_n, \\ -\frac{\boldsymbol{W}-\boldsymbol{W}_{\text{pre}}}{T_s} + 2\,\text{sym}(\mathbb{A}\overline{\boldsymbol{W}}) + 2\alpha\overline{\boldsymbol{W}} \preceq 2\,\text{sym}\left(\underline{\boldsymbol{g}}\boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top\right), \\ \overline{\boldsymbol{W}} = \mu\boldsymbol{W}, \end{cases} \tag{14}$$

Steady State Error Bound from Contraction Theory Based Control Design:

- Lumped uncertainty δ should be compensated to reduce the steady state error bound.

$$\lim_{t\to\infty}\|\boldsymbol{e}(t)\| = \sup_{\boldsymbol{x}\in\Omega_x,\boldsymbol{\pi},\boldsymbol{\pi}_n\in\Omega_\pi}\|\boldsymbol{g}\boldsymbol{\nu}+\boldsymbol{\delta}\|\frac{\sqrt{\chi}}{\alpha}. \tag{15}$$

Steady State Error Bound from Contraction Theory Based Control Design:

- Lumped uncertainty $\boldsymbol{\delta}$ should be compensated to reduce the steady state error bound.

$$\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = \sup_{\boldsymbol{x}\in\Omega_x,\boldsymbol{\pi},\boldsymbol{\pi}_n\in\Omega_\pi} \|\boldsymbol{g}\boldsymbol{\nu} + \boldsymbol{\delta}\| \frac{\sqrt{\chi}}{\alpha}. \tag{15}$$

Neural Network Integration:

- The auxiliary control input $\boldsymbol{\nu}$ is designed to compensate for the lumped uncertainty $\boldsymbol{\delta}$.
- NN $\boldsymbol{\Phi}(\boldsymbol{x}_n;\boldsymbol{\theta})$ with online learning capability is employed to estimate $\boldsymbol{\delta}$.
- $\boldsymbol{\theta}$: vectorized NN weight $\mathcal{W}$, ϵ: NN approximation error.

$$\boldsymbol{\nu}^* = \boldsymbol{\Phi}(\boldsymbol{x}_n;\boldsymbol{\theta}^*) + \epsilon, \text{ and } \boldsymbol{\nu} = \boldsymbol{\Phi}(\boldsymbol{x}_n;\widehat{\boldsymbol{\theta}}), \tag{16}$$

where

$$\boldsymbol{\Phi}(\boldsymbol{x}_n;\boldsymbol{\theta}) := \mathcal{W}_1^\top \phi(\mathcal{W}_0^\top \boldsymbol{x}_n), \tag{17}$$

Adaptation Law Derivation:

$$V(\boldsymbol{z}) := \frac{1}{2}\boldsymbol{e}^\top \boldsymbol{M}\boldsymbol{e} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_1^\top \boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{\theta}}_1, \tag{18}$$

- The adaptation law $\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\boldsymbol{\theta}}$ is derived using Lyapunov stability analysis.
- Uniformly ultimate boundedness (UBB) of the error dynamics is guaranteed.
- Contraction metric $\boldsymbol{M}$ is used as a weighting matrix for feedback tracking error $\boldsymbol{e}$.
- Stability analysis is omitted for brevity, see original paper for details.

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\boldsymbol{\theta}} := -\boldsymbol{\Gamma}\left(\frac{\partial \boldsymbol{\Phi}(\boldsymbol{x}_n; \widehat{\boldsymbol{\theta}})}{\partial \widehat{\boldsymbol{\theta}}}^\top \underline{\boldsymbol{g}}^\top \boldsymbol{M}\boldsymbol{e} + \sigma\widehat{\boldsymbol{\theta}}\right), \tag{19}$$

where $\boldsymbol{\Gamma}$ is the learning rate matrix and σ is σ-modification gain (regularization).

Simulation Environment:

- Numerical simulations are conducted using MATLAB.

Test Scenarios ($v_x = 110\,\mathrm{km\,h^{-1}}$):

- Step steering front steering and proportional rear steering are considered as desired input $\boldsymbol{u}_d$ to simulate reference model.
- Uniformly distributed random disturbances on each state with magnitudes of 10 and 20, respectively.

Vehicle Model:

- $m = 1463\,\mathrm{kg}$, $I_z = 1967.8\,\mathrm{kg\,m^2}$, $l_f = 1.2\,\mathrm{m}$, and $l_r = 1.6\,\mathrm{m}$.
- Assume heavy load on rear axle ($C_f = 78.4\,\mathrm{kN\,rad^{-1}}$, $C_r = 145.6\,\mathrm{kN\,rad^{-1}}$).

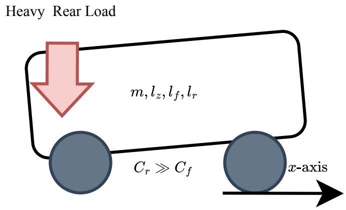


Figure: Simulation environment and test scenarios.

Table: Controllers for comparative study.

	Description	Control Input
(C_1) [blue]	Proposed controller	$\boldsymbol{u} = \boldsymbol{u}_d - \boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\boldsymbol{e} + \boldsymbol{\nu}$
(C_2) [cyan]	Conventional feedback controller based on contraction theory [10]	$\boldsymbol{u} = \boldsymbol{u}_d - \boldsymbol{R}^{-1}\underline{\boldsymbol{g}}^\top \boldsymbol{M}\boldsymbol{e}$
(C_3) [gray]	Desired controller	$\boldsymbol{u} = \boldsymbol{u}_d$

Simulation Results and Analysis

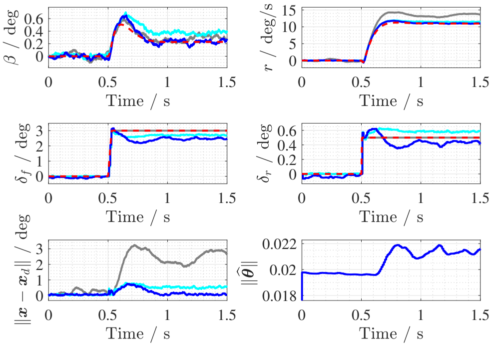


Figure: Validation results of (C_1) [blue line], (C_2) [cyan line] and (C_3) [gray line], and the desired trajectory [red dashed line].

- Using contraction theory, both (C_1) and (C_2) showed better tracking performance than (C_3).
- NN in (C_1) further improved the tracking performance by compensating for uncertainties.

Table: Tracking performances in L_2 norm.

	(C_1) [blue]	(C_2) [cyan]	(C_3) [gray]
$\sqrt{\int_0^T \|\boldsymbol{e}\|^2 \, \mathrm{d}t}$	9.768 (-87.34%)	17.469 (-77.36%)	77.150 (–)

MICLab
Mobility Intelligence and Control Laboratory

Mobility Intelligence and Control Laboratory

Conclusion:

Conclusion:

- This work presents a all-wheel steering system control based on contraction theory with neural networks.

Conclusion:

- This work presents a all-wheel steering system control based on contraction theory with neural networks.
- Contraction metric is obtained by solving LMI problem and used in feedback controller.

Conclusion:

- This work presents a all-wheel steering system control based on contraction theory with neural networks.
- Contraction metric is obtained by solving LMI problem and used in feedback controller.
- Neural networks are employed to compensate for uncertainties, ensuring closed-loop stability.

Conclusion:

- This work presents a all-wheel steering system control based on contraction theory with neural networks.
- Contraction metric is obtained by solving LMI problem and used in feedback controller.
- Neural networks are employed to compensate for uncertainties, ensuring closed-loop stability.
- Numerical simulations demonstrated the effectiveness of the proposed controller.

Thank you for your attention!

[1] H. Chu, D. Meng, S. Huang, M. Tian, J. Zhang, B. Gao, and H. Chen, "Autonomous high-speed overtaking of intelligent chassis using fast iterative model predictive control," *IEEE Transactions on Transportation Electrification*, vol. 10, no. 1, pp. 1244–1256, 2024. [Online]. Available: https://doi.org/10.1109/TTE.2023.3269602

[2] M. Akar, "Yaw rate and sideslip tracking for 4-wheel steering cars using sliding mode control," in *2006 IEEE Conference on Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control*, 2006, pp. 1300–1305. [Online]. Available: https://doi.org/10.1109/CACSD-CCA-ISIC.2006.4776830

[3] H.-M. Lv, N. Chen, and P. Li, "Multi-objective h ∞ optimal control for four-wheel steering vehicle based on yaw rate tracking," *Proceedings of the Institution of Mechanical Engineers, Part D*, vol. 218, no. 10, pp. 1117–1123, 2004. [Online]. Available: https://doi.org/10.1177/095440700421801006

[4] W. LOHMILLER and J.-J. E. SLOTINE, "On contraction analysis for non-linear systems," *Automatica*, vol. 34, no. 6, pp. 683–696, 1998. [Online]. Available: https://doi.org/10.1016/S0005-1098(98)00019-3

[5] H. Tsukamoto, S.-J. Chung, and J.-J. E. Slotine, "Contraction theory for nonlinear stability analysis and learning-based control: A tutorial overview," *Annual Reviews in Control*, vol. 52, pp. 135–169, 2021. [Online]. Available: https://doi.org/10.1016/j.arcontrol.2021.10.001

MIC Lab
Mobility Intelligence and Control Laboratory

[6] G. Cybenko, "Approximation by superpositions of a sigmoidal function," *Mathematics of Control, Signals, and Systems (MCSS)*, vol. 2, no. 4, pp. 303–314, Dec. 1989. [Online]. Available: http://dx.doi.org/10.1007/BF02551274

[7] S. S. Ge, C. C. Hang, T. H. Lee, and Z. Tao, *Stable adaptive neural network control*, ser. The International Series on Asian Studies in Computer and Information Science. New York, NY: Springer, Dec. 2010. [Online]. Available: https://doi.org/10.1007/978-1-4757-6577-9

[8] R. Rajamani, *Vehicle Dynamics and Control*, ser. Mechanical Engineering Series. Springer US, 2011. [Online]. Available: https://books.google.co.kr/books?id=cZJFDox4KuUC

[9] H. Tsukamoto and S.-J. Chung, "Learning-based robust motion planning with guaranteed stability: A contraction theory approach," *IEEE Robotics and Automation Letters*, vol. 6, no. 4, pp. 6164–6171, 2021. [Online]. Available: https://doi.org/10.1109/LRA.2021.3091019

[10] ——, "Neural contraction metrics for robust estimation and control: A convex optimization approach," *IEEE Control Systems Letters*, vol. 5, no. 1, pp. 211–216, 2021. [Online]. Available: https://doi.org/10.1109/LCSYS.2020.3001646