Physics-Informed Online Learning of Flux Linkage Model for Synchronous Machines

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***Abstract*—This study presents a physics-informed online learning method for modeling the flux linkages of synchronous machines (SMs). The approach trains neural networks (NNs) to learn the physical laws governing the flux linkages while adhering to the physical constraints inherent in the model. The learning rules are designed to satisfy the first-order optimality conditions with quadratic convergence. The proposed method can be used for both online state estimation and online model learning, ensuring fast convergence for local behavior and gradual, comprehensive convergence for global behavior, respectively.**

***Index Terms*—online learning, physics-informed learning, stator flux linkages, synchronous machines, system identification**

I. Introduction

A. Background

Synchronous machines (SMs) have been widely used in the industry. Permanent magnet synchronous machines (PMSMs) are particularly prevalent in automotive applications for vehicle traction [1], [2]. On the other hand, synchronous reluctance machines (SynRMs) are considered attractive alternatives to PMSMs due to their advantages, such as lower cost and the absence of magnets [3]. Recently, permanent magnet-assisted SynRMs (PMA-SynRMs), which combine the benefits of both PMSMs and SynRMs, have been adopted in Tesla's electric vehicles.

The electrical dynamics of synchronous machines (SMs) are described by first-order ordinary differential equations (ODEs) for the stator flux linkages [4]. Thus, accurate knowledge of the stator flux linkages is crucial in understanding the electrical behavior of the SM and designing a controller for the SM. For instance, model predictive control (MPC), one of the advanced control techniques based on optimization, utilizes information on the current values of the stator flux linkages or the inductances derived from them to predict the future behavior of the SM and select the optimal control action [5]. Accordingly, the accuracy of used information highly affects the performance of MPC.

Measuring the stator flux linkages inside the SM is challenging. Instead, the values of the stator flux linkages can be estimated at each steady-state operating point by using the equations derived from setting the time derivative terms in the ODEs to zero. The stator flux linkage maps can be obtained offline by an identification experiment over the entire operating range [6]. However, the map accuracy depends on the sophistication of the identification experiment at the expense of the cost and effort. Although the stator flux linkage maps are obtained accurately for the operating range over which the experiment is performed, they cannot deal with the parameter changes resulting from aging or abnormal operations, such as temperature increase or demagnetization. Thus, online estimation of stator flux linkages is crucial.

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B. Literature Review

Online estimation methods have been proposed to overcome the disadvantages of offline identification. The most straightforward way was to integrate the ODEs for the stator flux linkages in the stationary $\alpha-\beta$ frame to obtain their values [7]. However, the integration suffered errors due to inaccurate initial or input values. A high-pass filter was generally applied after the integration to remove the integration errors acting as DC offsets [8], but it also distorted the frequency response around and below the cutoff frequency. A method was proposed in [9] to recover the frequency response distortion by compensating for the difference between the frequency response of the pure integrator and the filter at the frequency of the SM rotation. However, this compensation only works well under steady-state conditions that the frequency-domain approach assumes, and it would degrade transient performance.

The stator flux linkages can also be estimated in the rotating $d-q$ frame. Many studies used the steady-state assumption, by which the d- and q-axis stator flux linkages were expressed as explicit functions of the stator voltages and currents [10]. This approach is simple and easy to implement, but the transient behavior of the SM was not considered at all. The high-frequency current injection has also been adopted for the stator flux linkage estimation [11] and has shown satisfactory steady-state performance. However, their transient performance was not sufficiently investigated. State observers, such as the Luenberger observer [12], sliding mode observer [13], and extended Kalman filter [14], have been widely investigated as stator flux linkage estimators, demonstrating satisfactory transient performance. However, most state observer-based estimators rely on prior knowledge of the synchronous motor (SM) electrical parameters, such as inductances, which can only be accurately identified after the stator flux linkages are determined. Consequently, their steady-state or transient performance can degrade when parameter information is inaccurate.

Recently, state observer-based online flux linkage estimators have been proposed, which do not require accurate knowledge of machine parameters but provide remarkable estimation performance. In [15], a disturbance observer-based flux linkage estimator (DOBFLE) was proposed, which could estimate the flux linkages without knowing the accurate value of the inductance matrix, with the help of the DOB compensating for the nonlinear disturbance term. An advanced $\alpha-\beta$ frame-based estimator was presented in [16], where integration errors were estimated by a linear state observer and compensated for in the time domain, which differed from using a frequency-domain approach. Both methods presented in [15] and [16] offered remarkable estimation performance even using inaccurate nominal machine parameters. However, they struggled to ensure exponential convergence during transient states, as these observers relied on steady-state assumptions for certain states. Their transient performance deteriorated significantly when nominal parameters deviated substantially from the true parameters.

C. Contributions

The literature review confirms that even state-of-the-art state observer-based approaches [15], [16] struggle to achieve exponential convergence during transient states. In response, this study proposes a novel data-driven approach that guarantees quadratic convergence based on the first-order optimality conditions [17] during online learning. The key difference between this approach and existing methods lies in the treatment of stator flux linkages. While existing methods treat the stator flux linkages as state variables to be estimated either in the frequency domain through signal processing or in the time domain via state observers, this approach views the stator flux linkages as functions of stator currents, which are identified through online learning of neural networks (NNs).

Very few studies have investigated data-driven approaches for the online identification of stator flux linkages, including [18], where the flux linkage model is represented by a neural network and trained online based on the error between the estimated and true flux linkage values. However, calculating the true flux linkage values is generally inaccurate, leading to suboptimal performance.

The contributions of the proposed method, and how it differs from existing data-driven approaches, are as follows:

- The flux linkage model is represented by NNs and identified through online learning, ensuring quadratic convergence based on the first-order optimality conditions.
- The proposed method incorporates physics-informed learning , where the electrical dynamics of the flux linkages (i.e., physical law) are learned without relying on the true flux linkage values. This is achieved by reformulating the ordinary differential equations (ODEs) for flux linkages into partial differential equations (PDEs).
- Physics-informed learning also imposes physical constraints on the flux linkage during the learning process, guiding the model and enhancing its robustness.
- The proposed physics-informed online learning method can be applied for both online state estimation and online model learning, ensuring fast convergence for local behavior and a more gradual, comprehensive convergence for global behavior, respectively.

D. Organization

The paper is outlined as follows: Section II provides preliminaries. Section III introduces the proposed method for physics-informed online learning of stator flux linkages. The simulation and experimental results are reported in Sections IV and **??**, respectively. Section V concludes with an outlook on future work.

II. Preliminaries

A. ODEs describing SM Electrical Dynamics

The electrical dynamics of SM are expressed in the rotating d-q reference frame by the following ODEs:

$$\frac{d\psi_d(i_d,i_q)}{dt} = -R_s i_d + w_r\psi_q(i_d,i_q) + v_d, \tag{1a}$$

$$\frac{d\psi_q(i_d,i_q)}{dt} = -R_s i_q - w_r\psi_d(i_d,i_q) + v_q, \tag{1b}$$

$$T_e = 1.5P\left(\psi_d(i_d,i_q)i_q - \psi_q(i_d,i_q)i_d\right), \tag{1c}$$

where ψ_d and ψ_q represent the d- and q-axis flux linkages, respectively; i_d and i_q are the d- and q-axis current, respectively; v_d and v_q represent the d- and q-axis voltage, respectively; R_s denotes the stator resistance R_s; w_r represents the electrical rotor speed; T_e is the output torque; and P is the number of pole pairs. Regarding the ODEs, the following assumptions are made:

- The d- and q-axis currents (i_d and i_q) and the electrical rotor speed (w_r) are known from direct measurements.
- The stator resistance (R_s) is assumed to be known.
- The inverter nonlinearity can be modeled and accurately compensated using the method presented in [19] (see Section III-E for more details). Therefore, the d- and q-axis voltages (v_d and v_q) are known from the reference voltages, which are corrected for nonlinearity.

The flux linkages are generally modeled as nonlinear functions of the currents. Consequently, the time derivatives of the flux linkages can be expressed as follows:

$$\begin{bmatrix} d\psi_d/dt \\ d\psi_q/dt \end{bmatrix} = \begin{bmatrix} \partial\psi_d/\partial i_d & \partial\psi_d/\partial i_q \\ \partial\psi_q/\partial i_d & \partial\psi_q/\partial i_q \end{bmatrix} \begin{bmatrix} di_d/dt \\ di_q/dt \end{bmatrix}, \tag{2}$$

where the partial derivatives are the d- and q-axis differential inductances, defined as $L_{dd} := \partial\psi_d/\partial i_d$ and $L_{qq} := \partial\psi_q/\partial i_q$, and the mutual differential inductances, defined as $L_{dq} := \partial\psi_d/\partial i_q$ and $L_{qd} := \partial\psi_q/\partial i_d$.

B. Interpreting ODEs as PDEs

The ODEs (1a) and (1b) can be reinterpreted as PDEs by substituting the terms $d\psi_d/dt$ and $d\psi_q/dt$ with their corresponding expressions from (2), and treating di_d/dt and

di_q/dt in (2) as time-varying parameters that can be measured. This yields the following PDEs:

$$\frac{di_d}{dt}\frac{\partial\psi_d}{\partial i_d}+\frac{di_q}{dt}\frac{\partial\psi_d}{\partial i_q}=-R_si_d+w_r\psi_q(i_d,i_q)+v_d, \tag{3a}$$
$$\frac{di_d}{dt}\frac{\partial\psi_q}{\partial i_d}+\frac{di_q}{dt}\frac{\partial\psi_q}{\partial i_q}=-R_si_q-w_r\psi_d(i_d,i_q)+v_q. \tag{3b}$$

This interpretation lays the foundation for employing physics-informed learning to perform online identification of the flux linkage model in SMs.

C. Constraints in Flux Linkages

Each synchronous machine (SM) exhibits distinct nonlinear characteristics for flux linkages; however, they share some common properties:

$$L_{dd}(i_d,i_q)>0, L_{qq}(i_d,i_q)>0 \tag{4}$$

while L_{dq} and L_{qd} can be either positive or negative. The primary distinction between different types of SMs lies in the values of ψ_d and ψ_q at zero currents:

$$\begin{bmatrix}\psi_d(0,0)\\ \psi_q(0,0)\end{bmatrix}=\begin{cases}\begin{bmatrix}\lambda_{pm} & 0\end{bmatrix}^T, \text{PMSM}, \text{PMa}-\text{SynRM}\\ \begin{bmatrix}0 & 0\end{bmatrix}^T, \quad \text{SynRM}\end{cases} \tag{5}$$

where $\lambda_{pm}>0$ represents the flux linkage due to the permanent magnets. Permanent magnet synchronous machines (PMSM) and permanent magnet-assisted synchronous reluctance machines (PMa-SynRM) exhibit nonzero flux linkages due to the presence of permanent magnets, while synchronous reluctance machines (SynRM), which do not use permanent magnets, have zero flux linkages at zero currents.

III. Physics-Informed Online Learning of Flux Linkage Model

A. Problem Formulation

In this study, the flux linakges are modeled using xxx NNs as follows:

$$\psi_d(i_d,i_q)=W_d^T\sigma_d(i_d,i_q)+\epsilon_d(i_d,i_q), \tag{6a}$$
$$\psi_q(i_d,i_q)=W_q^T\sigma_q(i_d,i_q)+\epsilon_q(i_d,i_q), \tag{6b}$$

where $W_d\in\mathbb{R}^m$ and $W_q\in\mathbb{R}^m$ represent the weight vectors for the d- and q-axis flux linkages, respectively; $\sigma_d\in\mathbb{R}^m$ and $\sigma_q\in\mathbb{R}^m$ are nonlinear activation functions for the d- and q-axes; and ϵ_d and ϵ_q denote the approximation errors of the NNs, which are bounded by sufficiently small positive values according to the universal approximation theorem [20]. The objective is to learn the weight vectors online and identify the estimated flux linkage model as follows:

$$\hat\psi_d(\hat W_d,i_d,i_q)=\hat W_d^T\sigma_d(i_d,i_q), \tag{7a}$$
$$\hat\psi_q(\hat W_q,i_d,i_q)=\hat W_q^T\sigma_q(i_d,i_q), \tag{7b}$$

where $\hat W_d\in\mathbb{R}^m$ and $\hat W_q\in\mathbb{R}^m$ are the estimated weight vectors for the d- and q-axis flux linkages, respectively, which are learned online. The estimated (mutual) differential inductances are expressed as:

$$\hat L_{dd}(\hat W_d,i_d,i_q):=\frac{\partial\hat\psi_d}{\partial i_d}=\hat W_d^T\frac{\partial\sigma_d(i_d,i_q)}{\partial i_d}, \tag{8a}$$
$$\hat L_{dq}(\hat W_d,i_d,i_q):=\frac{\partial\hat\psi_d}{\partial i_q}=\hat W_d^T\frac{\partial\sigma_d(i_d,i_q)}{\partial i_q}, \tag{8b}$$
$$\hat L_{qd}(\hat W_q,i_d,i_q):=\frac{\partial\hat\psi_q}{\partial i_d}=\hat W_q^T\frac{\partial\sigma_q(i_d,i_q)}{\partial i_d}, \tag{8c}$$
$$\hat L_{qq}(\hat W_q,i_d,i_q):=\frac{\partial\hat\psi_q}{\partial i_q}=\hat W_q^T\frac{\partial\sigma_q(i_d,i_q)}{\partial i_q}. \tag{8d}$$

The PDEs in (3) can be discretized using the Euler method with a sampling time T_s, where all partial derivatives are expressed in terms of inductance notations, resulting in the following discrete-time equations:

$$\begin{aligned}&L_{dd,k}\left(i_{d,k+1}-i_{d,k}\right)+L_{dq,k}\left(i_{q,k+1}-i_{q,k}\right)\\&=T_s\left(-R_si_{d,k}+w_{r,k}\psi_{q,k}+v_{d,k}\right)\end{aligned} \tag{9a}$$

$$\begin{aligned}&L_{qd,k}\left(i_{d,k+1}-i_{d,k}\right)+L_{qq,k}\left(i_{q,k+1}-i_{q,k}\right)\\&=T_s\left(-R_si_{q,k}-w_{r,k}\psi_{d,k}+v_{q,k}\right),\end{aligned} \tag{9b}$$

where the subscript k denotes the discrete time step; $L_{xy,k}=L_{xy}(i_{d,k},i_{q,k})$ with $x=d,q$ and $y=d,q$; and $\psi_{x,k}=\psi_x(i_{d,k},i_{q,k})$ with $x=d,q$. The errors in the discretized PDEs, resulting from substituting the estimated flux linkages $\hat\psi_d(\hat W_d,i_d,i_q)$ and $\hat\psi_q(\hat W_q,i_d,i_q)$, are given by:

$$\begin{aligned}e_{d,k+1}:=&\hat L_{dd,k}\left(i_{d,k+1}-i_{d,k}\right)+\hat L_{dq,k}\left(i_{q,k+1}-i_{q,k}\right)\\&-T_s\left(-R_si_{d,k}+w_{r,k}\hat\psi_{q,k}+v_{d,k}\right),\end{aligned} \tag{10a}$$

$$\begin{aligned}e_{q,k+1}:=&\hat L_{qd,k}\left(i_{d,k+1}-i_{d,k}\right)+\hat L_{qq,k}\left(i_{q,k+1}-i_{q,k}\right)\\&-T_s\left(-R_si_{q,k}-w_{r,k}\hat\psi_{d,k}+v_{q,k}\right).\end{aligned} \tag{10b}$$

Define the weight vector as $W:=\begin{bmatrix}W_d^T & W_q^T\end{bmatrix}^T\in\mathbb{R}^{2m}$ and the estimate of it as $\hat W:=\begin{bmatrix}\hat W_d^T & \hat W_q^T\end{bmatrix}^T\in\mathbb{R}^{2m}$. Let $\mathcal{K}$ denote the set of time steps selected for learning, which can be udpated in real-time (see Section III-D for more details). The objective function for minimizing the PDE errors is defined as

$$J_p(\hat W)=\sum_{k\in\mathcal{K}}\frac{1}{2}\left(\hat e_{d,k}^2+\hat e_{q,k}^2\right). \tag{11}$$

To enhance the capability of the PDE solutions for predicting measurable data (i.e., the d- and q-axis currents), the current prediction performance is also incorporated into the learning process. The current predictions are derived as follows:

$$\begin{aligned}&\begin{bmatrix}\hat i_{d,k+1}\\ \hat i_{q,k+1}\end{bmatrix}=\begin{bmatrix}i_{d,k}\\ i_{q,k}\end{bmatrix}\\&+T_s\begin{bmatrix}\hat L_{dd,k} & \hat L_{dq,k}\\ \hat L_{qd,k} & \hat L_{qq,k}\end{bmatrix}^{-1}\begin{bmatrix}-R_si_{d,k}+w_{r,k}\hat\psi_{q,k}+v_{d,k}\\ -R_si_{q,k}-w_{r,k}\hat\psi_{d,k}+v_{q,k}\end{bmatrix}.\end{aligned} \tag{12}$$

The objective function for minimizing the data prediction errors is defined as

$$J_d(\hat{W}) = \sum_{k\in\mathcal{K}} \frac{1}{2}\left(\left(i_{d,k}^{meas} - \hat{i}_{d,k}\right)^2 + \left(i_{q,k}^{meas} - \hat{i}_{q,k}\right)^2\right), \tag{13}$$

where $i_{d,k}^{meas}$ and $i_{q,k}^{meas}$ are the measured d- and q-axis currents, respectively.

Finally, the physics-informed online learning problem is formulated as follows:

$$\min_{\hat{W}} J(\hat{W}) = w_p J_p(\hat{W}) + w_d J_d(\hat{W}) \tag{14a}$$

subject to

$$c^{eq}(\hat{W}) = \hat{\psi}_q(\hat{W}_q, 0, 0) = 0, \tag{14b}$$

$$c_1^{in}(\hat{W}) = \hat{\psi}_d(\hat{W}_d, 0, 0) \geq \underline{\lambda}_{pm}, \tag{14c}$$

$$c_2^{in}(\hat{W}) = \hat{L}_{dd}(\hat{W}_d, i_d, i_q) \geq \underline{L}_{dd}, \tag{14d}$$

$$c_3^{in}(\hat{W}) = \hat{L}_{qq}(\hat{W}_q, i_d, i_q) \geq \underline{L}_{qq}, \tag{14e}$$

for $(i_d, i_q) = (), (), ...$

where w_p and w_d are the weighting factors for the objective functions J_p and J_d, respectively. The lower bounds $\underline{\lambda}_{pm} \geq 0$, $\underline{L}_{dd} > 0$, and $\underline{L}_{qq} > 0$ represent the minimum values for λ_{pm}, L_{dd}, and L_{qq}, respectively. The objective function (14a) is designed to optimize the learning of the PDEs (9), while minimizing the data prediction error. Constraints (14b) and (14c) enforce (5), and constraints (14d) and (14e) impose (4). If the lower bounds $\underline{\lambda}_{pm}$, $\underline{L}_{dd}$, and $\underline{L}_{qq}$ are unknown, they can be set to zero.

B. Learning Rules

The learning rules to solve the constrained optimization problem (14) are derived using the first-order optimality conditions [17]. The Lagrangian function is defined as follows:

$$L(\hat{W}, \lambda^{eq}, \lambda^{in}) := J(\hat{W}) - \lambda^{eq} c^{eq}(\hat{W}) - \sum_{j\in\mathcal{A}} \lambda_j^{in} c_j^{in}(\hat{W})$$

where λ^{eq} is the Lagrange multiplier for the equality constraint c^{eq}, λ_j^{in} is the Lagrange multiplier for the inequality constraint c_j^{in}, with $\lambda^{in} = [\lambda_1^{in}, \lambda_2^{in}, \lambda_3^{in}]^T$, and $\mathcal{A} := \{j \in \mathcal{I} \mid c_j^{in} \leq 0\}$ is the active set.

The learning rules for updating the estimated weight vector $\hat{W}$ and the Lagrangian multipliers λ^{eq} and $\lambda_{j\in\mathcal{A}}^{in}$ are as follows:

$$\hat{W}_n = \hat{W}_{n-1} - T_s \alpha \frac{\partial L(\hat{W}, \lambda^{eq}, \lambda^{in})}{\partial \hat{W}}, \tag{15a}$$

$$\lambda_n^{eq} = \lambda_{n-1}^{eq} + T_s \beta^{eq} c^{eq}(\hat{W}), \tag{15b}$$

$$\lambda_{j,n}^{in} = \max\left(\lambda_{j,n-1}^{in} + T_s \beta_j^{in} c_j^{in}(\hat{W}), 0\right), \forall j \in \mathcal{A}, \tag{15c}$$

where subscript n denotes the current time step, α is the learning rate for the weight vectors, and β^{eq} and β_j^{in} are the learning rates for the Largangian multipliers λ^{eq} and λ_j^{in}, respectively.

C. Convergence Analysis

Assuming that the sampling time T_s is sufficiently small, the learning rules (15) can be expressed in the continuous-time domain as follows:

$$\dot{\hat{W}} = -\alpha \frac{\partial L(\hat{W}, \lambda^{eq}, \lambda^{in})}{\partial \hat{W}}, \tag{16a}$$

$$\dot{\lambda}^{eq} = \beta^{eq} c^{eq}(\hat{W}), \tag{16b}$$

$$\dot{\lambda}_j^{in} = \beta_j^{in} c_j^{in}(\hat{W}), \forall j \in \mathcal{A}, \tag{16c}$$

where c_j^{in} is assumed to be instantly active during the update of λ_j^{in}, so that λ_j^{in} remains positive, allowing us to ignore the maximum operation in (15c).

Let the Lagrangian function $L(\hat{W}, \lambda^{eq}, \lambda^{in})$ serve as the loss function to be minimized during online learning. The following theorem provides the basis for convergence.

Theorem 1 *Suppose the sampling time T_s is sufficiently small, and c_j^{in} is instantly active during the online learning process. Then, the learning rules* (15) *update the estimated weight vector $\hat{W}$, as well as the Lagrangian multipliers λ^{eq} and λ_j^{in}, to satisfy the first-order optimality conditions for the loss function $L(\hat{W}, \lambda^{eq}, \lambda^{in})$, achieving quadratic convergence.*

PROOF Taking the time derivative of the loss function gives:

$$\dot{L} = \frac{\partial L}{\partial \hat{W}} \dot{\hat{W}} + \frac{\partial L}{\partial \lambda^{eq}} \dot{\lambda}^{eq} + \sum_{j\in\mathcal{A}} \frac{\partial L}{\partial \lambda_j^{in}} \dot{\lambda}_j^{in} \tag{17}$$

$$= -\alpha \left\| \frac{\partial L}{\partial \hat{W}} \right\|^2 - \beta^{eq} (c^{eq})^2 - \sum_{j\in\mathcal{A}} \beta_j^{in} \left(c_j^{in}\right)^2 \tag{18}$$

$$< 0. \tag{19}$$

This result demonstrates that the learning rules (15) ensure the satisfaction of the first-order optimality conditions with quadratic convergence.

Remark 1 In general, satisfying the first-order optimality conditions only guarantees the necessary conditions for optimality, meaning the solution could be a local minimum, a local maximum, or a saddle point. However, the proposed method is more likely to converge to a local minimum, aided by the constraints imposed during learning. Proper selection of the NNs and their initial values further ensures convergence to the local minimum.

D. Learning Modes

1) Online state estimation mode: In this mode, the proposed method estimates the current values of the flux linkages online. The NNs (6) are updated using data only from the current operating point (i.e., $\mathcal{K} = \{n\}$). The estimated weight vector must be updated quickly to adapt to changes in the operating point, using sufficiently high learning rates α, β^{eq}, and β_j^{in}. While the NNs may become overfitted to the current operating point and may not capture the global behavior of the flux linkages, this mode provides fast and accurate estimations, which are typically sufficient for conventional current controller design and one-step-ahead model predictive control (MPC) schemes.

2) Online model learning mode: In this mode, the proposed method is used to learn the global behavior of the flux linkages online by utilizing a data buffer that collects data from past

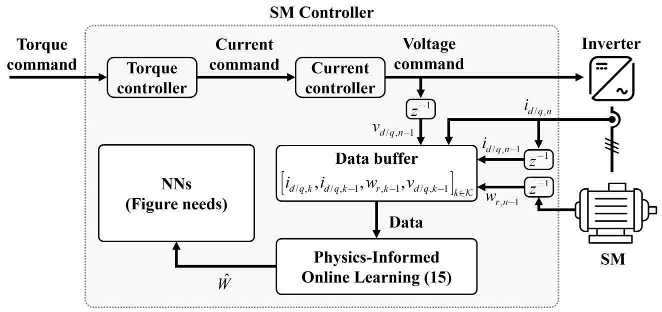


Fig. 1. Schematic diagram of the proposed physics-informed online learning method for the flux linkage model of SMs.

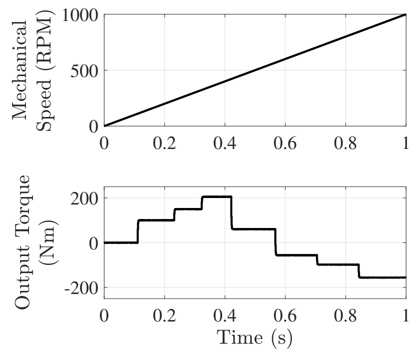


Fig. 2. Operating condition used for the validation of the online state estimation mode.

operating points (i.e., $\mathcal{K} = \{n, n-a, n-b, \dots\}$, where $b > a > 0$). The number of data points in the buffer should be at least equal to the number of parameters in the weight vectors W_d or W_q to prevent overfitting. Lower learning rates can be used to ensure stable learning of the global behavior. The flux linkage model learned in this mode can be applied in advanced algorithms, such as multi-horizon MPC, condition monitoring, fault diagnosis, and rotor temperature estimation.

E. Considerations on Implementation

1) Design of Activation Function: Any nonlinear functions, including commonly used activation functions for neural networks such as the hyperbolic tangent and radial basis function, can be considered as candidates for the activation functions in (6). However, the ideal activation function should inherently represent the behavior of the flux linkages and satisfy the relevant constraints (e.g., (14c) and (14d)). A prototype function that meets these criteria was proposed in [21]. Based on this, a suitable candidate for the activation function is defined as:

$$\sigma_{d(q)} = \begin{bmatrix} 1 & i_{d(q)} & \tanh\left(i_{d(q)}\right) & i_{q(d)}^2 & \ln\left(\cosh\left(i_{q(d)}\right)\right) \end{bmatrix}^T. \tag{20}$$

Each component of the activation function can be further scaled to ensure that the elements of the estimated weight vector $\hat{W}$ are evenly scaled, which may facilitate online learning and imporve robustness.

2) Inverter Nonlinearity and Angle Delay: The actual voltages synthesized by the inverter may differ from the reference voltages determined by the controller due to inverter nonlinearity. If the voltage vector error becomes significant enough to impact the learning results, it is recommended to compensate for the inverter nonlinearity. For systems using space vector pulse width modulation (SVPWM), the compensation method presented in [19] is applied. This technique analytically models the average inverter nonlinearity within the switching period and subtracts it from the reference voltages. For finite control set (FCS)-based control methods (e.g., FCS-MPC), the inverter nonlinearity is estimated using a switching device model, and this estimate is then subtracted from each voltage vector in the FCS.

The electrical rotor angle of the SM is used to perform the coordination transformation from the stationary frame to the $d-q$ frame. If there is a noticeable delay between the rotor angle measurement and the control action, it introduces a phase error to all vector variables in the $d-q$ frame. To address this, the angle delay must be identified and compensated. A straightforward but effective angle delay compensation method, described in [22], has been adopted in this study.

IV. Simulation Validation

A. Simulation Setup

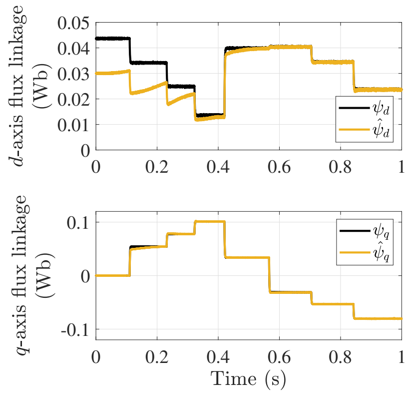
Simulation validation was conducted using MATLAB R2024a to verify the feasibility of the proposed method. The simulation environment was adapted from the `IPMSM Torque Control` example, incorporating the proposed method within the control framework. The interior PMSM (IPMSM) model in this environment had the following specifications: rated power = 385 W, rated torque = 6 Nm, base speed = 613 RPM, moment of inertia = 0.025 kg·m^2, $I_{max} = 5$ A, $V_{dc} = 250$ V, $P = 4$, $R_s = 0.1$, $L_d = 0.04$ H, and $L_q = 0.12$ H. The IPMSM drive was controlled using a built-in torque and current control algorithm.

The simulation was divided into two parts. The first part (see Section IV-B) validated the online state estimation mode of the proposed method. The IPMSM drive was tasked with tracking the torque command under the mechanical speed profile depicted in Fig. 2, while estimating the current flux linkages values in real time. The second part (see Section IV-C) evaluated the online model learning mode of the proposed method. The IPMSM drive operated under the Urban Dynamometer Driving Schedule (UDDS) cycle, a widely-used operating condition for automotive certification tests. The objective was to validate the method's ability to learn the global behavior of the flux linkage over the course of the given operating cycle.

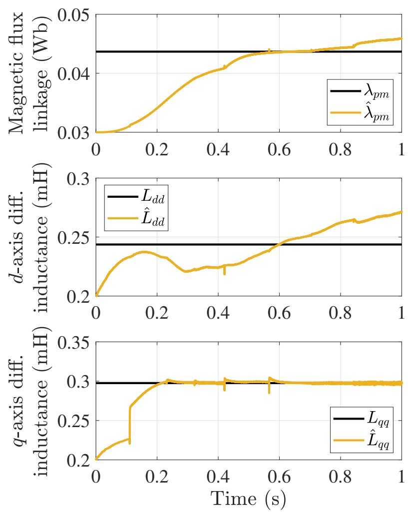
The activation functions for the NNs were designed as follows:

$$\sigma_d = \begin{bmatrix} 100 & i_d \end{bmatrix}^T, \tag{21a}$$

$$\sigma_q = \begin{bmatrix} 1 & i_q \end{bmatrix}^T. \tag{21b}$$

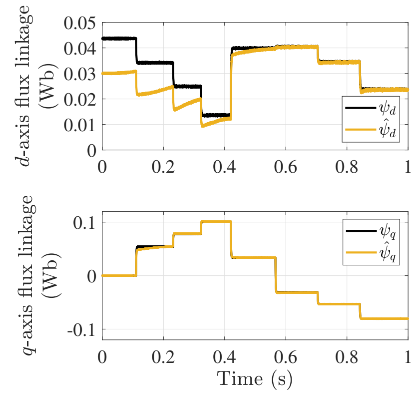


(a)

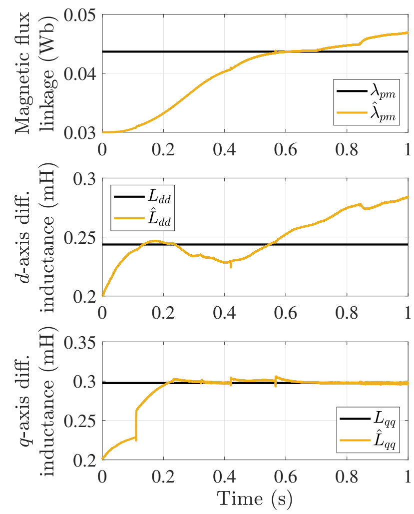


(b)

Fig. 3. Result of the online estimation mode with $w_p = 1$ and $w_d = 0.0000001$. (a) Estimation results of the d- and q-axis flux linkages. (b) Estimation results of the magnetic flux linkage, and d- and q-axis differential inductances.



(a)



(b)

Fig. 4. Result of the online estimation mode with $w_p = 0$ and $w_d = 0.0000001$. (a) Estimation results of the d- and q-axis flux linkages. (b) Estimation results of the magnetic flux linkage, and d- and q-axis differential inductances.

B. Validation of Online State Estimation Mode

C. Validation of Online Model Learning Mode

V. Conclusion

This study presented a physics-informed online learning method for modeling the flux linkages of SMs. To enable the use of physics-informed learning, the ODEs describing the electrical dynamics of the SM were reformulated as PDEs. The learning process was designed to minimize both the PDE errors, which encode the underlying physical laws, and the data prediction errors, to enhance prediction accuracy, all while incorporating physical constraints on the flux linkage model. The learning rules were shown to exhibit quadratic convergence. Simulation results using an 385-W IPMSM demonstrated the potential of the proposed method in both the online state estimation mode and the online model learning mode.

However, this study has a notable limitation. The inverter nonlinearity was assumed to be accurately compensated. Future research could address this by incorporating an additional neural network to simultaneously identify the inverter nonlinearity model alongside the flux linkage model.

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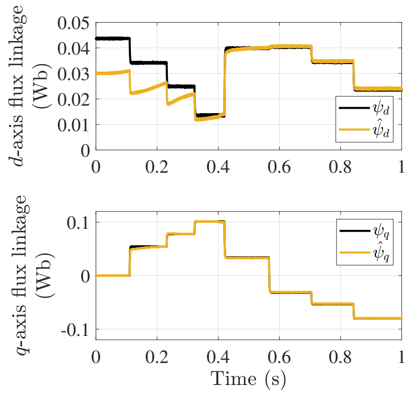
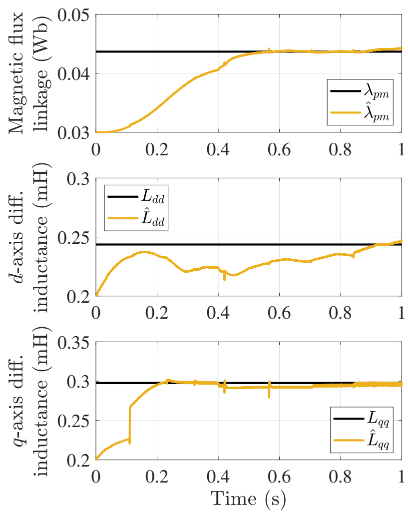


(a)



(b)

Fig. 5. Result of the online learning mode with $w_p = 1$ and $w_d = 0.0000001$. (a) Estimation results of the d- and q-axis flux linkages. (b) Estimation results of the magnetic flux linkage, and d- and q-axis differential inductances.

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