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Reverse path tracking control methods for tractor trailers using trailer steering system

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Abstract This paper presents a control strategy to enhance high-curvature trajectory tracking in tractor-trailer reverse maneuvers. This study incorporates rear-wheel steering control of the trailer, enabling more flexible movement and improved trajectory tracking performance. The control system consists of two components: a trailer rear-wheel steering controller, which aligns the trailer's rear end with the target path, and a tractor front-wheel steering controller, which stabilizes the hitch angle. Geometric analysis is used to derive methods for calculating both the target curvature toward the desired point and the trailer's current curvature. Additionally, a novel approach for determining lateral error is proposed, utilizing a local coordinate system with the trailer's rear end as the origin. The control system is designed using Simulink and simulated through TruckSim. The results demonstrate improved reverse path tracking performance when following a high-curvature trajectory with discontinuous curvature changes.

1. Introduction

The tractor-trailer is a large vehicle primarily used for freight transportation, playing a crucial role in efficiently carrying substantial loads. In the case of a tractor-trailer system, active lateral control of the entire vehicle is difficult when only the front wheel steering of the tractor is used. Recently, unlike tractor-trailers, trailer steering systems have been designed by introducing the steerable wheel on the last axle that allows the vehicle to bend, mitigating mobility constraints caused by its length. The angle at which this bending occurs is referred to as the hitch angle. While this hitch angle enhances maneuverability, it also introduces complex handling characteristics, requiring a higher level of skill to operate for the driver—particularly during reverse parking, which can be challenging even for experienced drivers. In recent years, the number of qualified tractor-trailer drivers has declined due to factors such as increased labor intensity, stricter environmental regulations, and more rigorous training and qualification standards aimed at accident prevention. As a result, various autonomous control techniques are emerging as potential replacements for skilled drivers.

Numerous control strategies have been explored for path tracking in tractor-trailer systems with tractor-steering systems or articulated structures without steering axles. Some studies have employed control theories widely used in other fields. For example, controllers incorporating model predictive control (MPC) and sliding mode control (SMC) have been developed using a dual closed-loop structure, demonstrating reduced turning radius and oscillations at the trailer's rear [1]. Additionally, Lyapunov-based controllers have been applied to tractor-trailer mobile robots for path tracking [2, 3]. A robust linear quadratic regulator (RLQR) has also been proposed to minimize the impact of payload weight variations [4]. Furthermore, a comparative study between MPC and linear quadratic regulator (LQR) methods was conducted to evaluate their effectiveness in high-performance path tracking [5].

In addition to conventional control approaches, novel methodologies have been introduced to address challenges associated with the nonlinear structure of off-axle vehicles. One such method involves assuming a ghost vehicle with an on-axle connection that mimics the actual vehicle's motion, simplifying the linearization process [6]. The verifiable tracking feedback (VTF)

method has been implemented for path tracking, demonstrating reduced tracking errors compared to PID and other control techniques [7]. Another approach introduced a control strategy based on the feasible domain of hitch angles during reversing, contributing to improved path-tracking performance [8]. A path planning and tracking framework for the general 2 trailer (G2T) has been proposed [9]. Instead of relying on complex nonlinear control systems, a method using tracking error to determine reference curvature and control the vehicle has been introduced [10]. Additionally, a control technique that accounts for slip effects has been developed to enhance tractor-trailer system performance in agricultural environments [11]. Another method stabilizes the hitch angle by calculating a target hitch angle based on path tracking error, enabling a relatively simple yet effective control approach [12].

To improve maneuverability in high-speed and curved environments, a path-tracking control technique utilizing trailer rear steering has been proposed to align the paths of lead and follow points [13]. Further research has focused on implementing forward and backward tracking, as well as velocity control, for tractor-trailer mobile robots [14]. Additionally, a study introduced two auxiliary modes for reversing path tracking: one where the driver manually sets the trailer's heading via a rotary knob and another that autonomously controls path tracking [15]. The multi-steering tractor-trailer mobile robot (MSTTMR) method has also been proposed, demonstrating superior maneuverability and flexibility compared to the single-steering tractor-trailer mobile robot (SSTTMR) [16].

Under the advancement of various control technologies, reverse path tracking control techniques using only tractor steering have also evolved in various directions. A method has been proposed to implement reverse path tracking for tractor-trailers by establishing constraints for jackknife avoidance and performing model predictive control (MPC) [17]. A sliding mode control (SMC) technique for reverse path tracking was also proposed, and its superiority was demonstrated by comparing its performance with a neural network (NN) controller [18]. Furthermore, by utilizing cascade control, jackknife phenomena could be actively prevented during reverse motion [19].

A key challenge in existing studies on tractor-trailer reverse path tracking is the instability of the hitch angle and the difficulty in following high-curvature paths with discontinuous curvature changes. While previous approaches have primarily focused on front-wheel steering control or complex nonlinear control methods, they often struggle with sharp turns and reversing maneuvers. This paper introduces a novel reverse path tracking control strategy that incorporates trailer rear-wheel steering, a departure from conventional methods that rely solely on tractor front-wheel steering. By actively controlling the trailer's rear wheels, the proposed method enhances maneuverability, improves tracking accuracy in high-curvature scenarios, and mitigates instability caused by abrupt curvature changes. The complexly coupled problem was addressed by decoupling it into two aspects: path tracking and hitch angle stabilization. This was achieved by independently controlling the tractor steering and trailer steering. Through kinematic analysis and PID control, it has been demonstrated that this approach, utilizing a simpler logic compared to conventional methods, can achieve superior performance. The effectiveness of the proposed approach is validated through simulations in Simulink and TruckSim, demonstrating precise reverse path tracking performance compared to existing control techniques.

2. Overall architecture

One of the biggest challenges in tractor-trailer control, as highlighted in previous studies, is nonlinearity. Therefore, rather than directly employing a dynamic model for control, a simpler control approach that stabilizes tracking error is often preferred [20]. In this paper, path tracking performance is evaluated based on the lateral error at the midpoint of the last axle, as it represents how well the trailer's rear follows the desired path. To achieve accurate path tracking using trailer rear-wheel steering, the tracking error serves as the input for the trailer steering controller. However, aligning the trailer's rear with the path through steering inputs inevitably induces a hitch angle. To mitigate this, the hitch error is used as the input for the tractor steering controller, ensuring stability and improved tracking performance.

Simulations will be conducted using TruckSim and MATLAB/Simulink. The tractor-trailer model used in the simulation is shown in Fig. 1. The controller is designed based on a tractor-trailer model with an on-axle connection. When represented as a bicycle model, its structure is depicted in Fig. 2. To ensure high path tracking performance, the tractor-trailer is programmed to reverse at a low speed of approximately 1 m/s. The overall

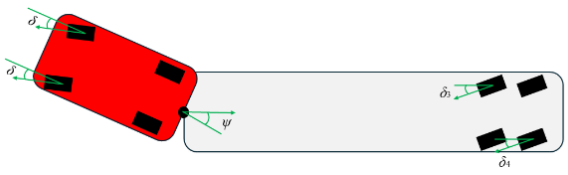


Fig. 1. Tractor-trailer system.

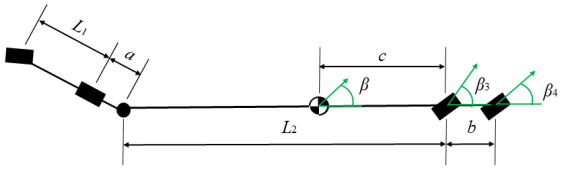


Fig. 2. Bicycle model of tractor-trailer system.

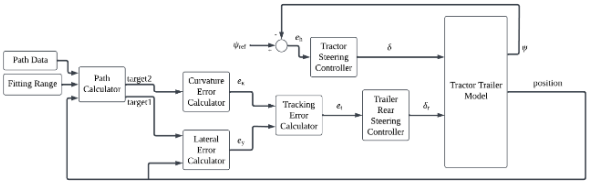


Fig. 3. Block diagram of the proposed controller.

structure of the controller is illustrated in Fig. 3.

In this study, 'target 1' represents the point on the desired path closest to the midpoint of axle 4, while 'target 2' is the goal point on the desired path at the rear of the trailer. The controller operates under the assumption that the desired path is pre-defined. The path data consists of global coordinates of points along the trajectory, which are provided to the controller in array format.

3. Tractor steering control

Excessive hitch angle in a tractor-trailer significantly affects driving stability, making it essential to continuously mitigate this issue through tractor steering control. The input to the tractor steering controller is defined to match the hitch angle with the reference hitch angle. Consequently, the tractor steering system generates appropriate steering inputs to stabilize the hitch angle by preventing excessive hitch angle and fluctuation, ensuring improved tracking performance and overall vehicle stability. Thus, the hitch angle error, e_h, and steering input of the tractor is defined as follows:

$$e_h = \psi_{ref} - \psi \tag{1}$$

$$\delta = K_P \cdot e_h + K_I \cdot \int e_h dt + K_D \cdot \dot{e}_h \tag{2}$$

where, K_P, K_I, and K_D mean the proportional, integral, and derivative gain for the e_h.

Here, ψ_{ref} and ψ denote the reference hitch angle and the actual hitch angle, respectively, with e_h representing their difference. Although the primary purpose of the tractor steering system is to control the hitch angle towards zero by setting a zero reference, maintaining the hitch angle too close to zero while exclusively managing lateral error can lead to significant deviation from the desired path when the tractor is reversing on a curved trajectory. Therefore, maintaining a certain degree of hitch angle is necessary for effective path tracking. Thus, it is crucial to properly tune K_P, K_I, and K_D to allow for some hitch deviation. Regarding K_I, its primary function is to eliminate steady-state error and bring the hitch error close to zero. However, if the gain is set too high, particularly if K_I is too high, the hitch error may be overcorrected, causing the tractor-trailer to reverse in an excessively straightened state. It is often preferable to set it very low or omit it entirely to avoid excessive correction.

4. Trailer rear steering control

4.1 Trailer steering input decision

The trailer rear-wheel steering input is determined by the following Eqs. (3) and (4).

$$\delta_3 = \delta_{ack,3} + \delta_{add} \tag{3}$$

$$\delta_4 = \delta_{ack,4} + \delta_{add} \tag{4}$$

Here, δ_{ack} represents the body slip angle at the wheels of the respective axle. As the body slip angle is a difficult value to measure in practice, δ_{ack} for axles 3 and 4 is defined under two assumptions. First, the body slip angle can be estimated from the trailer's mass center. Second, the trailer is a rigid body. The body slip angle can be estimated using IMU-based heuristics [21]. Since this paper assumes the tractor-trailer is moving at a constant velocity, it is also presumed that the longitudinal and lateral velocities can be estimated. Therefore, the controller in this paper will be designed under the premise that these longitudinal and lateral velocities are given.

$$\delta_{ack,3} = -\tan^{-1}\frac{vy_2 + c\cdot\dot{\theta}_2}{vx_2} \tag{5}$$

$$\delta_{ack,4} = -\tan^{-1}\frac{vy_2 + (b+c)\dot{\theta}_2}{vx_2} \tag{6}$$

Here, vx_2, vy_2, and θ_2 mean the longitudinal and lateral velocities and the yaw angle of the trailer.

As mentioned earlier, the tracking error will be used as the input for trailer steering. Since lateral error and curvature error significantly affect path tracking, these two factors will be used to compute the tracking error [17]. The tracking error e_t is determined as follows:

$$e_t = K(K_y e_y + K_\kappa e_\kappa) \tag{7}$$

Here, the gain K can be either negative or positive depending on the signs assigned to e_y and e_κ. The methods for calculating e_y and e_κ will be explained in Secs. 4.2 and 4.3. The calculated e_t is then multiplied by the PID gain to obtain δ_{add} as follows:

$$\delta_{add} = K_{P,r}\cdot e_t + K_{I,r}\cdot\int e_t dt + K_{D,r}\cdot\dot{e}_t \tag{8}$$

4.2 e_y decision

In Fig. 4, point P represents the closest point on the path, target1. e_y denotes the lateral coordinate of point P within the trailer's local coordinate system, where axle 4 serves as the origin. The calculation process is as follows:

$$\gamma = \frac{\pi}{2} + \theta_2 \tag{9}$$

$$\begin{aligned} e_y &= \begin{bmatrix}\cos\gamma & \sin\gamma\end{bmatrix}\begin{bmatrix}x_P - x_T \\ y_P - y_T\end{bmatrix} \\ &= (x_P - x_T)\cos\gamma + (y_P - y_T)\sin\gamma \end{aligned} \tag{10}$$

Here, T denotes the coordinates of axle 4, P represents the coordinates of target 1, and γ is defined as the angle between the positive global x-axis and $\overline{TP}$. By substituting the calculated e_κ and e_y into Eq. (7), e_t can be determined. Subsequently, δ_{add}

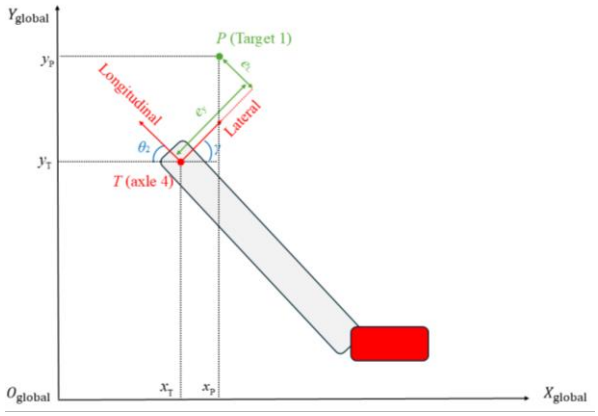
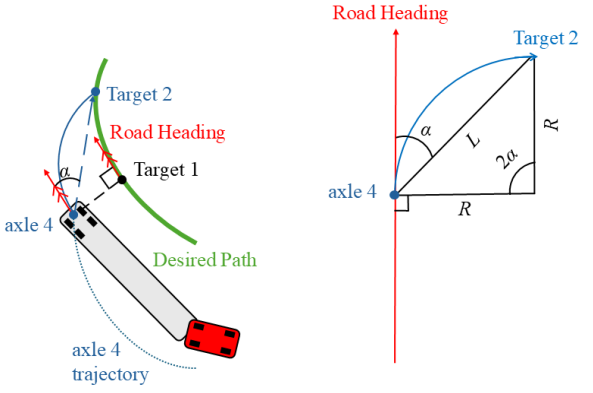


Fig. 4. Local coordinate system of trailer axle 4.



(a) Global view (b) Local view

Fig. 5. Method to calculate reference curvature.

is computed using Eq. (8). Finally, this value is substituted into Eqs. (3) and (4) to obtain the rear-wheel steering input.

4.3 e_κ decision

To determine the curvature error, it is essential to compute both the target curvature and the current curvature of the trailer. Instead of using the curvature of the predefined path, this approach utilizes the curvature of the trajectory toward the selected target 2 from the previous process. This method helps minimize steady-state error, improving overall tracking accuracy.

The path toward the target 2 is shown in Fig. 5(a). To compute the target curvature, the road heading must first be determined. This is obtained by differentiating $f(t)$ and $g(t)$ with respect to t. The trajectory toward the target 2 is then defined as a circular path that passes through the target 2 while remaining tangent to the straight line in the direction of the road heading at axle 4, as illustrated in Fig. 5(b).

By performing a geometric analysis, the following equation can be derived as follows:

$$\kappa_{ref} = \frac{2\sin\alpha}{L} \tag{11}$$

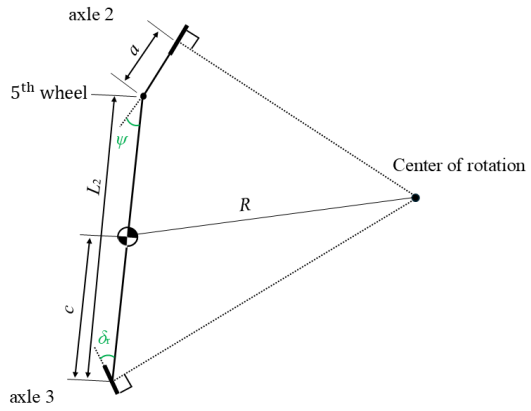


Fig. 6. Bicycle model of trailer.

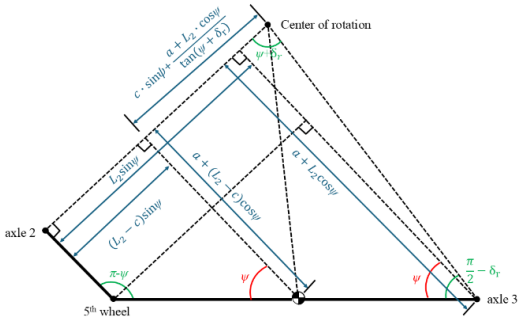


Fig. 7. Geometrical analysis of the bicycle model.

It is important to note that when α is negative, the curvature is also assigned a negative value. In this study, the curvature of a right-turning curve is defined as positive, while the curvature of a left-turning curve is defined as negative. The current curvature of the tractor-trailer system is determined by the kinematic bicycle model as shown in Fig. 6. For simplification in both the equations and the bicycle model, axle 4 has been excluded from the representation. A geometric analysis of this model is illustrated in Fig. 7.

Based on the geometric relationship given in Fig. 7, the curvature of the tractor-trailer system is defined as follows:

$$\kappa = \left[\left\{c\sin\psi + \frac{a+L_2\cos\psi}{\tan(\psi+\delta_r)}\right\}^2 + \left\{a+(L_2-c)\cos\psi\right\}^2\right]^{-\frac{1}{2}} \tag{12}$$

Similarly, if the path curves to the right relative to the trailer's reverse direction, it is assigned a positive curvature. The curvature error e_κ can be calculated using Eq. (11) as follows:

$$e_\kappa = \kappa_{ref} - \kappa \tag{13}$$

4.4 Desired path fitting

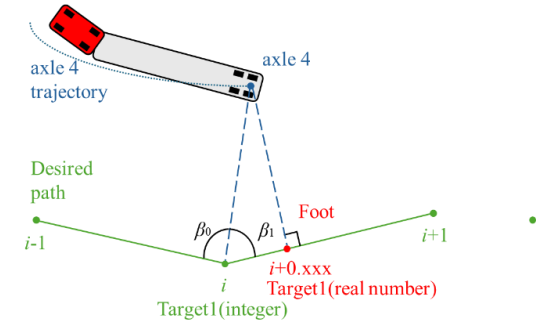
As mentioned earlier, the path data is received as an array. If this discrete data is used directly for path tracking, the control

input values may fluctuate, leading to instability. To address this, the proposed controller fits the path coordinates into a continuous curve over a specified range, creating a desired path. To generate the desired path, the system first identifies target1, the closest point on the path to axle 4, and determines its corresponding integer index. This index is then converted into a real number index, and the points within the fitting range are used to fit a single continuous curve.

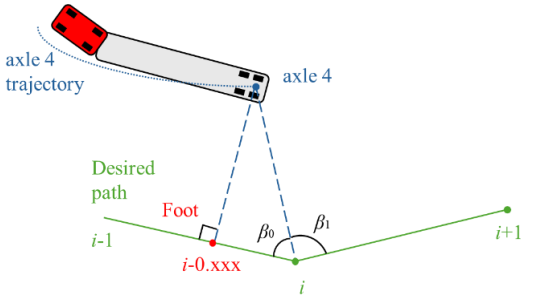
The term "fitting range" refers to the segment of data used for curve fitting, which is an adjustable parameter defined by the user. A wider fitting range results in a more accurately drawn curve but may slow down processing speed. The equation of the fitted curve is expressed as $x = f(t)$ and $y = g(t)$ with respect to the parameter index t. This formulation allows for direct computation of global coordinates when the index is known.

4.5 Real number indexing

Since path data consists of a discrete array of coordinate values, the index of any point on the path is inherently an integer. However, using integer indices as target1 can cause fluctuating steering inputs, reducing the precision of path tracking. To achieve smoother and more accurate steering, a real number index is introduced.



(a) Case of $\beta_0 > \beta_1$



(b) Case of $\beta_0 < \beta_1$

Fig. 8. A foot of the perpendicular to the discrete waypoint array.

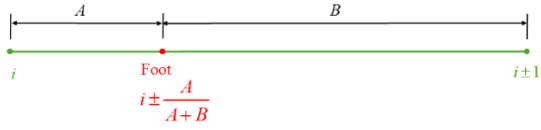


Fig. 9. Conceptual diagram to show the calculation of real number index.

Let i represent the integer index of the target 1 on the path to axle 4. As shown in Fig. 8, lines are drawn connecting points i-1 to i and i to i+1. A perpendicular is projected from axle 4 onto these line segments. The real number index is then calculated using a simple length ratio, as illustrated in Fig. 9.

However, following conditions must be considered:

1) If both β_0 and β_1 are obtuse angles, the perpendicular falls outside the segment, and the integer index i should be used directly as the real number index.
2) If β_0 equals β_1, or if either angle is 90°, the real number index remains i.

A specific offset value is added to the calculated real number index to determine the target 2's index, which serves as the goal for path tracking. This point corresponds to target 2, as mentioned earlier. The offset value should remain constant—a smaller value improves path-tracking performance. However, if it is too small, the system may fail to track the path effectively. Therefore, it is essential to fine-tune this parameter for each controller to achieve optimal tracking accuracy.

5. Simulation results

In this study, path tracking performance is evaluated based on lateral error, hitch angle, trailer rear-wheel steering angle, and tractor front-wheel steering angle. The tractor model '2A Euro Cab Over' and the trailer model '15.5 t Trailer, Tandem, Leaf (NHTSA)' were used. A road friction coefficient of 0.85 was applied, and the tire model used was '3000 kg rating, 510 mm radius'. As mentioned in Sec. 4.2, the lateral error is defined with respect to the center of axle 4 in the trailer. The desired path used in the simulation consists of a 40-meter straight segment, followed by an S-shaped path composed of two semicircular arcs, and concluding with another 40-meter straight segment. This path design is chosen to assess the system's ability to accurately follow both straight and curved trajectories, as well as its robustness in handling abrupt curvature changes. In this study, transition point where the path curvature abruptly shifts from positive to negative is referred to the "abrupt curvature change zone." In all simulations, the tractor-trailer follows a given S-shaped path in reverse, and the vehicle speed is set to 1 m/s.

5.1 Results of existing controller

To show the effectiveness of the trailer steering system, the results of the existing front-wheel steering control of the tractor are summarized in Figs. 10 and 11. Since this study focuses on evaluating path-tracking performance at large curvatures, the desired path is defined with radii ranging from 40 meters to the minimum radius that can be followed by the existing controller. Here, the radius refers to the radius of the two semicircles that form part of the desired path. To perform a realistic simulation, constraints for the maximum steering angle and angular rate are

applied to prevent unrealistically rapid changes in steering angles. The tractor's front-wheel steering angle is limited to ±36°, with an angular velocity constraint of ±4°/s.

The trajectories of the simulation results are summarized in Fig. 10. As shown in Fig. 10, the vehicle follows the desired path with a radius of 40 m without any abnormality. However, in the case of following a 30 m radius path, it can be seen that vibration occurs at the curvature discontinuity point where the semicircle path continues, and the hitch angle does not converge to a steady state. This phenomenon intensified when following a path with a radius of 20 m, and the trailer vibrated from the moment it entered the semicircle path from the straight path.

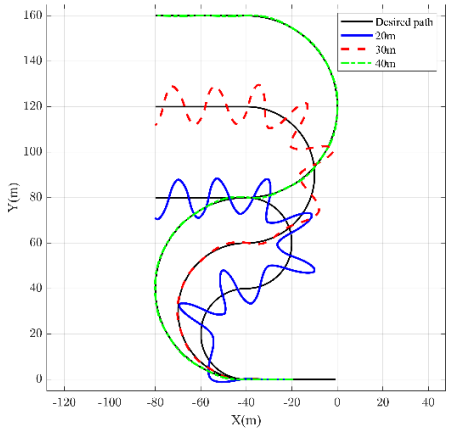
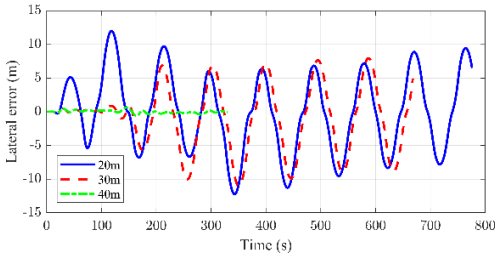
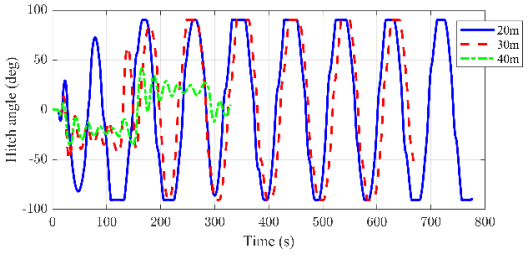


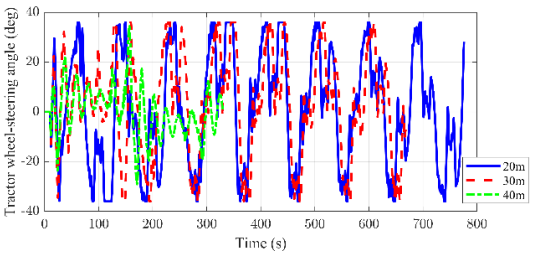
Fig. 10. Desired path and trajectories of the center of axle 4 with existing controller.



(a) Lateral error



(b) Hitch angle



(c) Tractor wheel steering angle

Fig. 11. Simulation results of existing controller.

The time histories of the simulation are summarized in Fig. 11. The termination time of each simulation varies depending on the radius of the desired path, as the total path length changes accordingly, while the tractor-trailer speed remains constant. In an ideal path-tracking scenario, the termination time should exhibit an approximately linear relationship with the radius. However, at radii of 30 and 20 meters, the lateral error is significantly larger than at 40 meters as shown in Fig. 11(a). Thus, the termination time sharply increases as curvature increases.

Specifically, the 40-meter radius results in an average lateral error below 0.5 meters, indicating satisfactory tracking performance. Ideally, in the abrupt curvature change zone, the signs of the tracking data should also flip in sync with the curvature change of the desired path. At a 40-meter radius, this condition is approximately satisfied, suggesting that the ideal path following is achieved. However, while the first semicircle of a 30-meter radius is followed with relatively good performance, the system fails to maintain stability in the abrupt curvature change zone, resulting in significant oscillations. At a 20-meter radius, the large curvature of the desired path prevents effective tracking even in the first semicircle, indicating that the controller cannot adequately handle such high curvature paths. While the tractor-trailer does not completely deviate from the path, effective path tracking is realistically achievable only up to a 40-meter radius.

The hitch angle history given in Fig. 11(b) also shows the oscillation at radii of 30 and 20 meters. Ideally, the hitch angle should be maintained at a certain bent position to effectively follow the path. However, when the curvature suddenly reverses, as in an S-shaped path, the hitch angle should also reverse rapidly. This abrupt shift induces excessive control inputs, leading to significant oscillations that destabilize the system. Additionally, Fig. 11(c) appears to exhibit similar oscillations in front steering angle input.

5.2 Results of proposed controller

The following results given in Figs. 12 and 13 are obtained from the simulation using the proposed controller, which incorporates trailer rear-wheel steering. The trailer rear-wheel steering angle is also subject to limitations, with a range of ±50° and an angular velocity limit of ±4°/s. The tractor's front-wheel steering angle is constrained identically to the existing controller.

The proposed controller demonstrates stable performance not only within a 30-meter radius but also at greater curvatures as given in Fig. 12. While the existing controller achieved meaningful path tracking only up to a 40-meter radius, the proposed controller maintains acceptable performance even at a 15-meter radius. Additionally, it exhibits robust control without significant oscillations in the abrupt curvature change zone.

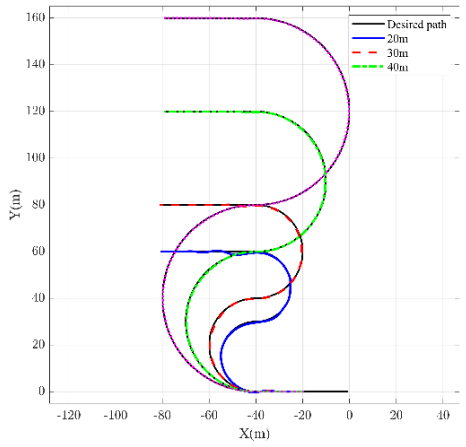
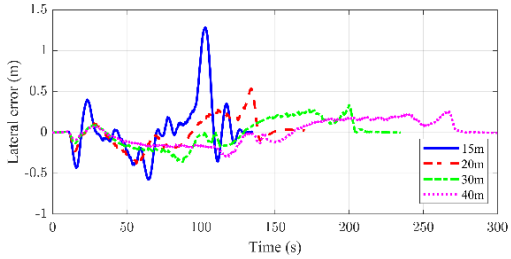
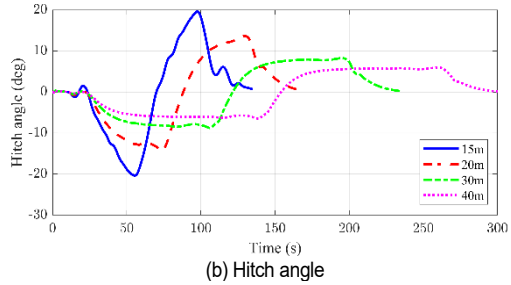


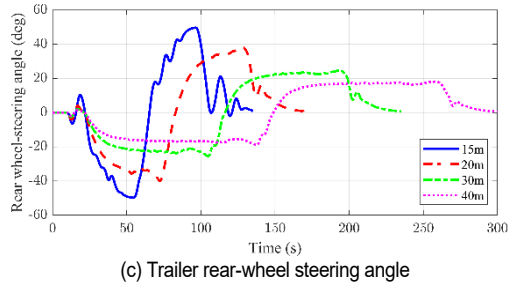
Fig. 12. Desired path and trajectories of the center of axle 4 with the proposed controller.



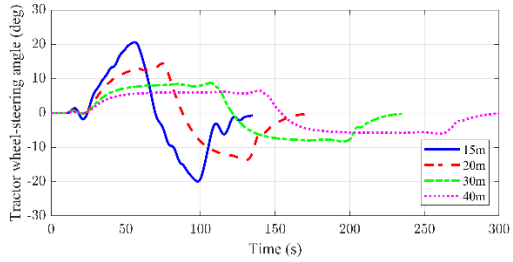
(a) Lateral error



(b) Hitch angle



(c) Trailer rear-wheel steering angle

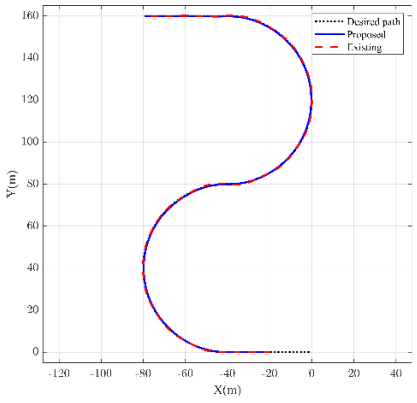


(d) Tractor wheel steering angle

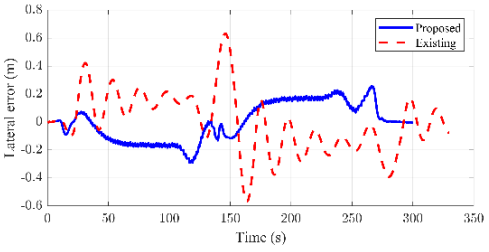
Fig. 13. Simulation results of existing controller.

5.3 Comparison between existing and proposed controllers

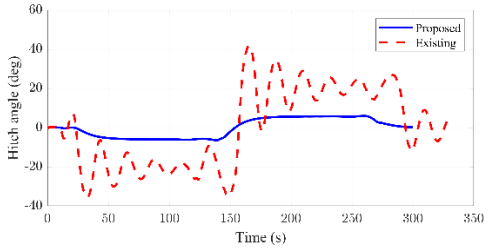
The performance comparison between the existing and the proposed controller was performed for the case of following a path with a radius of 40 m. This is because the existing controller was able to follow up to a path with a radius of 40 m. In Fig. 14, both controllers demonstrate satisfactory path-tracking performance. However, the existing controller exhibits oscillations in time history, whereas the proposed controller maintains minimal oscillation and demonstrates high stability.



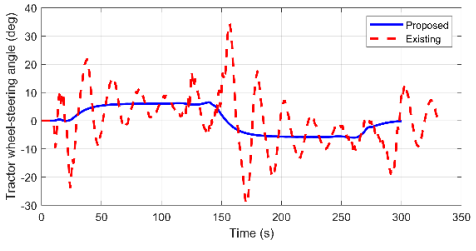
(a) Desired path and axle 4 trajectories



(b) Lateral error



(c) Hitch angle



(d) Tractor wheel steering angle

Fig. 14. Comparison of data at 40 m radius.

The primary difference between the two controllers lies in their approach to hitch stabilization. The existing controller relies solely on tractor steering, meaning it must perform both path tracking and hitch stabilization through the tractor's front-wheel steering. In contrast, the proposed controller, equipped with both tractor and trailer steering, enables path tracking through the trailer's rear-wheel steering while simultaneously stabilizing the hitch with the tractor's steering. Since hitch stability is critical in tractor-trailer systems, this represents a major advantage of the proposed controller.

6. Conclusions

In this paper, we proposed a path-tracking controller for reverse driving for high-curvature paths by utilizing both the tractor's front wheels and the trailer's rear wheels. A desired path, which is defined as discrete waypoints, is fitted separately for x and y coordinates using parameters. In addition, the real number index is proposed based on linear interpolation to prevent sudden changes in the feedback signal caused by waypoint movement. The input of the tractor's front steering is defined to track the desired path by matching the hitch angle with the path heading. Then, the additional control input of the trailer steering is determined to regulate the lateral and curvature errors, which are calculated based on the rear axle of the trailer. The simulation study showed that the proposed algorithm exhibited robustness against abrupt curvature changes and effectively stabilized the hitch when following the path in reverse direction.

For future works, an adaptation mechanism for the tractor and trailer dimensions can improve the robustness and usability of the proposed algorithm. In particular, a tractor-trailer is a commercial vehicle and can have various sizes and specifications, so an adaptive mechanism that can automatically match the control gain with the vehicle's specifications is thought to be effective. In addition, expansion to reverse path tracking is required in a situation where a plurality of trailers are connected. In general, when two or more trailers are connected, it is virtually impossible to drive backward only with the steering of a tractor. Therefore, a trailer with a steering function should be used, and a controller for effectively controlling it should be proposed together.

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Nomenclature

Ψ : Hitch angle
Ψ_{ref} : Reference hitch angle
δ : Tractor steering angle
δ_3 : Axle 3 wheel steering angle
δ_4 : Axle 4 wheel steering angle
L_1 : Wheelbase of the tractor
L_2 : Wheelbase of the trailer
a : Distance between axle 2 and hitch
b : Distance between axle 3 and axle 4
c : Distance between the trailer mass center and axle 3
β : Body slip angle of the trailer
β_3 : Body slip angle of axle 3 of the trailer
β_4 : Body slip angle of axle 4 of the trailer
e_h : Heading error of the tractor
e_y : Lateral error of the trailer
e_κ : Curvature error of the trailer
θ_1 : Tractor yaw angle
θ_2 : Trailer yaw angle
vx_2 : Velocity in x-direction at trailer mass center
vy_2 : Velocity in y-direction at trailer mass center

References

[1] Z. Liu, M. Yue, L. Guo and Y. Zhang, Trajectory planning and robust tracking control for a class of active articulated tractor-trailer vehicle with on-axle structure, *European Journal of Control*, 54 (2020) 87-98.
[2] P. Kassaeiyan, K. Alipour and B. Tarvirdizadeh, A full-state trajectory tracking controller for tractor-trailer wheeled mobile robots, *Mechanism and Machine Theory*, 150 (2020) 103872.
[3] A. Astolfi, P. Bolzern and A. Locatelli, Path-tracking of a tractor-trailer vehicle along rectilinear and circular paths: A Lyapunov-based approach, *IEEE Transactions on Robotics and Automation*, 20 (1) (2004) 154-160.
[4] F. M. Barbosa, L. B. Marcos, M. M. da Silva, M. H. Terra and V. G. Junior, Robust path-following control for articulated heavy-duty vehicles, *Control Engineering Practice*, 85 (2019) 246-256.
[5] M. Yue, X. Hou, R. Gao and J. Chen, Trajectory tracking control for tractor-trailer vehicles: A coordinated control approach, *Nonlinear Dynamics*, 91 (2018) 1061-1074.
[6] P. Bolzern, R. M. DeSantis, A. Locatelli and D. Masciocchi, Path-tracking for articulated vehicles with off-axle hitching, *IEEE Transactions on Control Systems Technology*, 6 (4) (1998) 515-523.
[7] Y. Gao, D. Cao and Y. Shen, Path-following control by dynamic virtual terrain field for articulated steer vehicles, *Vehicle System Dynamics*, 58 (10) (2020) 1528-1552.
[8] G. Xia, M. Zhao, X. Tang, S. Wang and L. Zhao, Linear reversing control of semi-trailer trains based on hitch angle stable and feasible domain, *Control Engineering Practice*, 104 (2020) 104625.
[9] O. Ljungqvist, N. Evestedt, D. Axehill, M. Cirillo and H. Pettersson, A path planning and path-following control framework for a general 2-trailer with a car-like tractor, *Journal of Field Robotics*, 36 (8) (2019) 1345-1377.
[10] Z. Leng and M. A. Minor, Curvature-based ground vehicle control of trailer path following considering sideslip and limited steering actuation, *IEEE Transactions on Intelligent Transportation Systems*, 18 (2) (2016) 332-348.

[11] V. T. Huynh, J. Katupitiya, N. M. Kwok and R. Eaton, Derivation of an error model for tractor-trailer path tracking, *Proceedings of the 2010 IEEE International Conference on Intelligent Systems and Knowledge Engineering*, Hangzhou, China (2010) 60-66.

[12] C. Pradalier and K. Usher, A simple and efficient control scheme to reverse a tractor-trailer system on a trajectory, *Proceedings of the 2007 IEEE International Conference on Robotics and Automation*, Rome, Italy (2007) 2208-2214.

[13] B. Jujnovich and D. Cebon, Path-following steering control for articulated vehicles, *Journal of Dynamic Systems, Measurement, and Control*, 135 (3) (2013) 031006.

[14] M. Sampei, T. Tamura, T. Itoh and M. Nakamichi, Path tracking control of trailer-like mobile robot, *Proceedings of the 1991 IEEE/RSJ International Workshop on Intelligent Robots and Systems (IROS'91)*, Osaka, Japan (1991) 193-198.

[15] M. Werling, P. Reinisch, M. Heidingsfeld and K. Gresser, Reversing the general one-trailer system: Asymptotic curvature stabilization and path tracking, *IEEE Transactions on Intelligent Transportation Systems*, 15 (2) (2013) 627-636.

[16] J. Yuan, F. Sun and Y. Huang, Trajectory generation and tracking control for double-steering tractor–trailer mobile robots with on-axle hitching, *IEEE Transactions on Industrial Electronics*, 62 (12) (2015) 7665-7677.

[17] M. Fan, M. Yue, H. Zhang and Z. Liu, Anti-jackknife reverse perpendicular parking control of tractor-trailer vehicle via MPC technique, *Proceedings of the IEEE International Conference on CYBER Technology in Automation, Control, and Intelligent Systems*, Suzhou, China (2019) 1138-1143.

[18] Y. Bin Salamah, Sliding mode controller for autonomous tractor-trailer vehicle reverse path tracking, *Applied Sciences*, 13 (21) (2023) 11998.

[19] A. Aldughaiyem, Y. Bin Salamah and I. Ahmad, Control design and assessment for a reversing tractor–trailer system using a cascade controller, *Applied Sciences*, 11 (22) (2021) 10634.

[20] C. Pradalier and K. Usher, Robust trajectory tracking for a reversing tractor trailer, *Journal of Field Robotics*, 25 (6-7) (2008) 378-399.

[21] D. Selmanaj, M. Corno, G. Panzani and S. M. Savaresi, Vehicle sideslip estimation: A kinematic based approach, *Control Engineering Practice*, 67 (2017) 1-12.



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